HYDRAULIC EXPERIMENTS.

LOWELL

HYDRAULIC EXPERIMENTS,

BEING A

SELECTION FROM EXPERIMENTS

ON

HYDRAULIC MOTORS,

ON THE

FLOW OF WATER OVER WEIRS, AND IN CANALS OF UNIFORM RECTANGULAR SECTION AND OF SHORT LENGTH.

MADE AT

LOWELL, MASSACHUSETTS.

BY

JAMES B. FRANCIS,
CIVIL ENGINEER, FELLOW OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES ETC.

BOSTON:
LITTLE, BROWN AND COMPANY.
1855.

Entered according to Act of Congress in the year 1855, by
JAMES B. FRANCIS,
in the Clerk's Office of the District Court of the District of Massachusetts.

CAMBRIDGE:
ALLEN AND FARNHAM, PRINTERS.

STONE AND SMART STEREOTYPERS.

TABLE OF CONTENTS.

INTRODUCTION.

PART I.

EXPERIMENTS ON HYDRAULIC MOTORS.

Number of the Article.		Page
	Experiments upon the Tremont Turbine,	1
1–17.	Introduction	1
18–35.	Description of the Turbine,	7
36–47.	Description of the Apparatus used in the Experiments,	14
48–53.	Mode of conducting the Experiments,	19
54–74.	Description of Table II., containing the Experiments upon the Turbine at the Tremont Mills,	25
75–82.	Description of the Diagram representing the Experiments,	36
83–88.	Path described by a Particle of Water in passing through the Wheel,	39
89–98.	Rules for proportioning Turbines,	44
99–109.	Experiments on a Model of a Centre-Vent Water-Wheel, with Straight Buckets,	55
110–119.	Experiments on the Power of a Centre-Vent Water-Wheel, at the Boott Cotton-Mills,	61

PART II.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, AND IN SHORT RECTANGULAR CANALS.

	Experiments on the Flow of Water over Weirs,	71
120–125.	Introduction,	71
126–185.	Experiments made at the Tremont Turbine, on the Flow of Water over Weirs,	73

136 Experiments on the Flow of Water over Weirs, made at the Centre-Vent Wheel for moving the Guard Gates of the Northern Canal, ... 96
137. Experiments on the Effect produced on the Flow of Water over Weirs, by the Height of the Water on the Down-stream Side, ... 99
138–147. Experiments on the Flow of Water over Weirs, made at the Lower Locks, Lowell, . 103
148–159. Description of Table XIII., containing the Details of the Experiments on the Flow of Water over Wiers made at the Lower Locks, Lowell, in October and November, 1852, 112
160–163. Comparison of the proposed Formula with the Results obtained by previous Experimenters, 126
164. Precautions to be observed in the application of the proposed Formula, . . . 133
165–166. Experiments on the Discharge of Water over a Dam of the same Section as that erected by the Essex Company, across the Merrimack River at Lawrence, Massachusetts, . 136
167–175. Experiments to ascertain the Effect of taking the Depths upon a Weir, by means of Pipes opening near the Bottom of the Canal, ... 137

176–187 A METHOD OF GAUGING THE FLOW OF WATER IN OPEN CANALS, OF UNIFORM RECTANGULAR SECTION, AND OF SHORT LENGTH, ... 143

TABLES.

PART I.

Number of the Table — Page
I. Weight of a Cubic Foot of pure Water, at different Temperatures, ... 20
II. Experiments upon the Turbine at the Tremont Mills, in Lowell, Massachusetts, . . . 32
III. Successive Steps in the Calculation for the Path of the Water in Experiment 30 on the Tremont Turbine, ... 41
IV. Table for Turbines of different Diameters, operating on different Falls, ... 53
V. Experiments on a Model of a Centre-Vent Water-Wheel, ... 58
VI. Experiments on the Boott Centre-Vent Water-Wheel, ... 66
VII. Successive Steps in the Calculation for the Path of the Water in Experiment 30, on the Boott Centre-Vent Water-Wheel, ... 70

PART II.

VIII. Experiments made at the Tremont Turbine, for the purpose of testing the Method of reducing the Depths on the Weir to a uniform Fall, ... 83
IX. Experiments made at the Tremont Turbine, which were repeated under identical circumstances, 84
X. Experiments on the Flow of Water over Weirs, made at the Tremont Turbine, . . 88

XI. Experiments on the Flow of Water over Weirs, made at the Centre-Vent Wheel for moving the Guard Gates of the Northern Canal, at Lowell, Massachusetts, ... 98

XII. Experiments on the Effect produced on the Flow of Water over Weirs, by the Height of the Water on the Downstream Side, ... 102

XIII. Experiments on the Flow of Water over Weirs, made at the Lower Locks, Lowell, in October and November, 1852, ... 122

XIV. Comparison of the proposed Formula with the Experiments of Poncelet and Lesbros, . 128

XV. Comparison of the proposed Formula with the Experiments of Castel, ... 130

XVI. Experiments on the Discharge of Water over a Dam of the same Section as that erected by the Essex Company, across the Merrimack River at Lawrence, Massachusetts, . . 137

XVII. Experiments made at the Lower Locks, to determine the Corrections to be applied to the Readings of the Hook Gauges, ... 140

XVIII. Experiments to ascertain the Effect of taking the Depths upon a Weir, by means of Pipes opening near the Bottom of the Canal, ... 142

XIX. Details of the Measurement in the Flume, ... 151

XX. Comparison of the Measurements in the Flume, with the Measurements at the Weirs, . 153

INTRODUCTION.

The northern regions of the United States of North America, probably possess a greater amount of water-power than any other part of the world of equal extent, and the active and inventive genius of the American people, combined with the very high price of labor, has had a powerful influence in bringing this power into use. Nevertheless, the water-power is so vast, compared with the population, that only a small portion of it has, up to this time, been applied to the purposes of man. It was estimated, not long since, that the total useful effect derived from water-power in France, was about 20,000 horse-power. An amount of power far exceeding this, is already derived from the Merrimack River and its branches, in Massachusetts and New Hampshire. What must be the amount of the population and wealth of the Northern States, when the other rivers that water them are equally improved?

One of the earliest and most successful efforts to bring into use, in a systematic manner, one of the larger water-powers, was made at Lowell in Massachusetts; where, in 1821, a number of farms situated near Pawtucket Falls on the Merrimack River, were purchased by several capitalists of Boston, who obtained a charter from the State of Massachusetts under the name of *The Merrimack Manufacturing Company*. In 1826, the property was transferred to the *Proprietors of the Locks and Canals on Merrimack River*, a corporation chartered in 1792 for the purpose of improving the navigation of the Merrimack River. Previously to the transfer, the Merrimack Manufacturing Company had erected a dam of about 950 feet in length, at the head of Pawtucket Falls, and had also enlarged the Pawtucket Canal, which was originally constructed, previously to the year 1800, by the Proprietors of the Locks and Canals on Merrimack River, for the purposes of navigation Subsequently to the enlargement, however, this canal has been used both for purposes of navigation, and to supply water to the wheels of numerous manufacturing establishments.

The dam at the head of Pawtucket Falls, in the ordinary state of the river, deadens the current of the river for about 18 miles, forming, in low water, a reservoir of about 1120 acres; this extensive reservoir is of great value in very low stages of the river, as it affords space for the accumulation of the flow of the river during the night, when the manufactories are not in operation. This accumulation is subsequently drawn off, together with the natural flow of the river, during the usual working hours

The total fall of the Merrimack River at Pawtucket Falls, in ordinary low water, is about 35 feet, of which about 2 feet is lost in consequence of the descent in the canals, leaving a net fall of about 33 feet. About $\frac{1}{8}$ of the water is used on the entire fall, and the remainder is used twice over, on falls of about 14 and 19 feet respectively The water-power has been granted by the Proprietors of the Locks and Canals on Merrimack River, in definite quantities called *Mill Powers*, which are equivalent to a gross power of a little less than 100 horse-power each. Grants have been made to eleven manufacturing companies, who have an aggregate capital, somewhat exceeding thirteen millions of dollars. Thus, to the Merrimack Manufacturing Company, there have been granted $24\frac{2}{3}$ mill powers, each of which consists of the right to draw, for 15 hours per day, 25 cubic feet of water per second on the entire fall. Up to this time, there have been granted at Lowell $139\frac{11}{30}$ mill powers, or a total quantity of water equal to 3595.933 cubic feet per second. A large portion of this water is used on turbines of a very superior description, and nearly all the remainder, on breast wheels of good construction, a portion of which, however, do not use quite the whole of the fall on which they are placed. We may, however, assume that, upon an average, a useful effect is derived, equal to $\frac{2}{3}$ of the total power of the water expended. Calling the fall 33 feet, and the weight of a cubic foot of water 62.33 pounds, we shall have for the effective power derived from the water-power granted by the Proprietors of the Locks and Canals on Merrimack River at Lowell,

$$\frac{3595.933 \times 62.33 \times 33 \times \frac{2}{3}}{550} = 8965.4 \text{ horse-power.}$$

In consequence of the success attending the improvement of the water-power at Lowell, several other extensive water-powers in New England have been brought into use in a similar manner. Some of these undertakings have been quite successful, whilst with others, as yet only partially developed, the success has not been so decided.

The great abundance of water-power in this country has had a strong tendency to encourage its extravagant use; the machines used in the manufactories are usually great consumers of power; the ability of a machine to turn off the greatest quantity of work with the least manual labor, and in the *least time*, has been the point mainly considered; and whether it required a greater or less amount of power, has been a secondary consideration

The engineering operations connected with the water-power at Lowell, have frequently demanded more definite information on certain points in hydraulics, than was to be found in any of the publications relating to that science; and hence has arisen the necessity, from time to time, of making special experiments to supply the required information. Whenever such emergencies have arisen, the officers who have the general care of the interests of the several corporations, with a liberality founded on enlarged views of the true interests of the bodies they represent, have always been willing to defray such expenses as were necessary, in order that the experiments might be made in a satisfactory manner.

The experiments recorded in the following pages, are a selection from those made by the author, in the discharge of his duty, as the Engineer of the Corporations at Lowell. They may be divided into two classes, namely, *First*, those on hydraulic motors, and, *second*, those on the flow of water over weirs, and in short rectangular canals. Combined with the description of the experiments, there are also given some other investigations, which may appear somewhat out of place, but which, from their utility or novelty, will be found interesting to many persons who have cultivated the science of hydraulics.

The unit of length adopted in this work, is the English foot according to a brass standard measure made by Cary of London, now in the possession of the Lowell Machine Shop.

HYDRAULIC EXPERIMENTS.

PART I.

EXPERIMENTS ON HYDRAULIC MOTORS.

EXPERIMENTS UPON THE TREMONT TURBINE.

1. UNTIL within a few years, the water-wheels in use in the principal manufacturing establishments in New England, were what are there generally called *breast wheels*, sometimes known also by the name of *pitch back wheels*. They are the same in principle as the *overshot-wheel*, the useful effect being produced, almost entirely, by the simple weight of the water in the buckets, and differing only from the *overshot-wheel* in this, that the water is not carried entirely over the top of the wheel, but is let into the buckets near the top, but on the opposite side from that adopted for the overshot-wheel. An apron, fitting as closely as practicable to the wheel, is used to prevent the water leaving the buckets, until it reaches very nearly the bottom of the wheel.

In Lowell, these wheels have been constructed principally of wood, many of them of very large dimensions. Those in the mills of the Merrimack Manufacturing Company, for instance, are thirty feet in diameter, with buckets twelve feet long. Four of the mills belonging to this company, have two such wheels in each of them.

Until the year 1844, the breast wheel, as above described, was considered here the most perfect wheel that could be used. Much prejudice existed here, as elsewhere, against the reaction wheels; a great number of which had, however, been used throughout the country, in the smaller mills, and with great advantage; for, although they usually gave a very small effect in proportion to the quantity of water expended, their cheapness, the small space required for them, their greater velocity, being less impeded by backwater, and not requiring expensive wheelpits of masonry, were very important considerations; and in a country where water power is so much more abundant than capital, the economy of money was generally of greater importance than the saving of water.

A vast amount of ingenuity has been expended by intelligent millwrights, on these wheels; and it was said, several years since, that not less than three hundred patents relating to them, had been granted by the United States Government. They continue, perhaps as much as ever, to be the subject of almost innumerable modifications. Within a few years, there has been a manifest improvement in them, and there are now several varieties in use, in which the wheels themselves are of simple forms, and of single pieces of cast-iron, giving a useful effect approaching sixty per cent. of the power expended.

2. The attention of American engineers was directed to the improved reaction water-wheels in use in France and other countries in Europe, by several articles in the Journal of the Franklin Institute; and in the year 1843, there appeared in that journal, from the pen of Mr. Ellwood Morris, an eminent engineer of Pennsylvania, a translation of a French work, entitled, *Experiments on water-wheels having a vertical axis, called turbines, by Arthur Morin, Captain of Artillery, etc. etc.* In the same journal, Mr. Morris also published an account of a series of experiments, by himself, on two turbines constructed from his own designs, and then operating in the neighborhood of Philadelphia.

The experiments on one of those wheels, indicate a useful effect of seventy-five per cent. of the power expended, a result [illegible] good as that claimed for the practical effect of the best overshot-wheels, which had, heretofore, in this country, been considered unapproachable, in their economical use of water.

3. In the year 1844, Uriah A. Boyden, Esq., an eminent hydraulic engineer of Massachusetts, designed a turbine of about seventy-five horse-power, for the Picking House of the Appleton Company's cotton-mills, at Lowell, in Massachusetts, in which wheel, Mr. Boyden introduced several improvements, of great value.

The performance of the Appleton Company's turbine, was carefully ascertained by Mr. Boyden, and its effective power, exclusive of that required to carry the wheel itself, a pair of bevel gears, and the horizontal shaft carrying the friction pulley of a Prony dynamometer, was found to be seventy-eight per cent. of the power expended.

4. In the year 1846, Mr. Boyden superintended the construction of three turbines of about one hundred and ninety horse-power each, for the same company. By the terms of the contract, Mr Boyden's compensation depended upon the performance of the turbines, and it was stipulated that two of them should be tested. The contract also contained the following clause, "and if the mean power derived from

these turbines be seventy-eight per cent. of the power of water expended, the Appleton Company to pay me twelve hundred dollars for my services, and patent rights for the apparatus for these mills; and if the power derived be greater than seventy-eight per cent, the Appleton Company to pay me, in addition to the twelve hundred dollars, at the rate of four hundred dollars for every one per cent. of power, obtained above seventy-eight per cent." In accordance with the contract, two of the turbines were tested, a very perfect apparatus being designed by Mr. Boyden for the purpose, consisting, essentially, of a Prony dynamometer to measure the useful effects, and a weir to gauge the quantity of water expended.

5. A great improvement in the mode of conducting hydraulic experiments was here adopted, in making each set of observations continuous, the time of each observation being noted; thus, the observer who noted the height of the water above the wheel, recorded regularly, say every thirty seconds, the time and the height; and so with the other observers, the recorded times furnishing the means of afterwards identifying simultaneous observations.

6. The observations were put into the hands of the author, for computation, who found that the mean maximum effective power of the two turbines tested, was eighty-eight per cent of the power of the water expended.

According to the terms of the contract, this made the compensation for engineering services, and patent rights for these three wheels, amount to fifty-two hundred dollars, which sum was paid by the Appleton Company without objection.

7. These turbines have now been in operation about eight years, and their performance has been, in every respect, entirely satisfactory The iron-work for these wheels was constructed by Messrs. Gay and Silver, at their machine shop at North Chelmsford, near Lowell; the workmanship was of the finest description, and of a delicacy and accuracy altogether unprecedented in constructions of this class

8. These wheels, of course, contained Mr. Boyden's latest improvements, and it was evidently for his pecuniary interest that the wheels should be as perfect as possible, without much regard to cost. The principal points in which one of them differs from the constructions of Fourneyron, are as follows.

9. *The wooden flume, conducting the water immediately to the turbine, is in the form of an inverted truncated cone, the water being introduced into the upper part of the cone, on one side of the axis of the cone (which coincides with the axis of the turbine) in such a manner, that the water, as it descends in the cone, has a gradually increasing velocity, and a spiral motion; the horizontal component of the spiral motion being in the direction of the motion of the wheel.* This horizontal motion is derived from the necessary velocity with which the water enters the truncated cone; and the arrangement is such that, if perfectly proportioned, there would be no loss of power between the nearly still water in the principal

penstock and the guides or leading curves near the wheel, except from the friction of the water against the walls of the passages. It is not to be supposed that the construction is so perfect as to avoid all loss, except from friction; but there is, without doubt, a distinct advantage in this arrangement over that which had been usually adopted, and where no attempt had been made to avoid sudden changes of direction and velocity.

10. *The guides, or leading curves, are not perpendicular, but a little inclined backwards from the direction of the motion of the wheel, so that the water, descending with a spiral motion, meets only the edges of the guides.* This leaning of the guides has also another valuable effect; when the regulating gate is raised only a small part of the height of the wheel, the guides do not completely fulfil their office of directing the water, the water entering the wheel more nearly in the direction of the radius, than when the gate is fully raised; by leaning the guides, it will be seen that the ends of the guides, near the wheel, are inclined, the bottom part standing further forward, and operating more efficiently in directing the water, when the gate is partially raised, than if the guides were perpendicular.

11. In Fourneyron's constructions, a garniture is attached to the regulating gate, and moves with it, for the purpose of diminishing the contraction; this, considered apart from the mechanical difficulties, is probably the best arrangement; to be perfect, however, theoretically, this garniture should be of different forms for different heights of gate; but this is evidently impracticable.

In the Appleton Turbine, the garniture is attached to the guides, the gate (at least the lower part of it) being a simple thin cylinder. By this arrangement, the gate meets with much less obstruction to its motion than in the old arrangement, unless the parts are so loosely fitted as to be objectionable; and it is believed that the coefficient of effect, for a partial gate, is proportionally as good as under the old arrangement.

12. *On the outside of the wheel is fitted an apparatus named, by Mr. Boyden, the Diffuser.* The object of this extremely interesting invention, is to render useful a part of the power otherwise entirely lost, in consequence of the water leaving the wheel with a considerable velocity. It consists, essentially, of two stationary rings or discs, placed concentrically with the wheel, having an interior diameter a very little larger than the exterior diameter of the wheel; and an exterior diameter equal to about twice that of the wheel; the height between the discs, at their interior circumference, is a very little greater than that of the orifices in the exterior circumference of the wheel, and at the exterior circumference of the discs, the height between them is about twice as great as at the interior circumference; the form of the surfaces connecting the interior and exterior circumferences of the discs, is gently rounded, the first elements of the curves, near the interior circumferences, being nearly horizontal. There is con-

sequently, included between the two surfaces, an aperture gradually enlarging from the exterior circumference of the wheel, to the exterior circumference of the diffuser. When the regulating gate is raised to its full height, the section, through which the water passes, will be increased by insensible degrees, in the proportion of one to four, and if the velocity is uniform in all parts of the diffuser at the same distance from the wheel, the velocity of the water will be diminished in the same proportion; or its velocity on leaving the diffuser, will be one fourth of that at its entrance. By the doctrine of living forces, the power of the water in passing through the diffuser must, therefore, be diminished to one sixteenth of the power at its entrance. It is essential to the proper action of the diffuser, that it should be entirely under water; and the power rendered useful by it, is expended in diminishing the pressure against the water issuing from the exterior orifices of the wheel, and the effect produced, is the same as if the available fall under which the turbine is acting, is increased a certain amount. It appears probable that a diffuser of different proportions from those above indicated, would operate with some advantage without being submerged. It is nearly always inconvenient to place the wheel entirely below low-water-mark; up to this time, however, all that have been fitted up with a diffuser, have been so placed; and, indeed, to obtain the full effect of a fall of water, it appears essential, even when a diffuser is not used, that the wheel should be placed below the lowest level to which the water falls in the wheelpit, when the wheel is in operation.

The action of the diffuser depends upon similar principles to that of diverging conical tubes, which, when of certain proportions, it is well known, increase the discharge; the author has not met with any experiments on tubes of this form, discharging under water, although, there is good reason to believe, that tubes of greater length and divergency would operate more effectively under water, than when discharging freely in the air; and that results might be obtained, that are now deemed impossible by most engineers.

Experiments on the same turbine, with and without a diffuser, show a gain in *the coefficient of effect*, due to the latter, of about three per cent. By the principles of living forces, and assuming that the motion of the water is free from irregularity, the gain should be about five per cent. The difference is due, in part at least, to the unstable equilibrium of water, flowing through expanding apertures; this must interfere with the uniformity of the velocities of the fluid streams, at equal distances from the wheel.

18. *Suspending the wheel from the top of the vertical shaft, instead of running it on a step at the bottom.* This had been previously attempted, but not with such success as to warrant its general adoption. It has been accomplished with complete success by Mr Boyden, whose mode is, to cut the upper part of the shaft into a series of necks,

and to rest the projecting parts upon corresponding parts of a box. A proper fit is secured by lining the box, which is of cast-iron, with babbitt metal, a soft metallic composition consisting, principally, of tin; the cast-iron box is made with suitable projections and recesses to support and retain the soft metal, which is melted and poured into it, the shaft being at the same time in its proper position in the box. It will readily be seen that a great amount of bearing surface can be easily obtained by this mode, and also, what is of equal importance, it may be near the axis; the lining metal, being soft, yields a little if any part of the bearing should receive a great excess of weight. The cast-iron box is suspended on gimbals, similar to those usually adopted for mariners' compasses and chronometers, which arrangement permits the box to oscillate freely in all directions, horizontally, and prevents, in a great measure, all danger of breaking the shaft at the necks, in consequence of imperfections in the workmanship, or in the adjustments. Several years' experience has shown, that this arrangement, carefully constructed, is all that can be desired; and that a bearing thus constructed, is as durable, and can be as readily oiled, and taken care of, as any of the ordinary bearings in a manufactory.

14. The buckets are secured to the crowns of the wheel in a novel, and much more perfect manner, than had been previously used; the crowns are first turned to the required form, and made smooth; by ingenious machinery devised for the purpose, grooves are cut with great accuracy in the crowns, of the exact curvature of the buckets; mortices are cut through the crowns, in several places in each groove; the buckets, or floats, are made with corresponding tenons, which project through the crowns, and are riveted on the bottom of the lower crown, and on the top of the upper crown; this construction gives the requisite strength and firmness, with buckets of much thinner iron than was necessary under any of the old arrangements; it also leaves the passages through the wheel entirely free from injurious obstructions.

15 Mr. Boyden has also designed a large number of turbines for different manufacturing establishments in New England, many of them under contracts similar to that with the Appleton Company, and has accumulated a vast number of valuable experiments and observations upon them, which, it is to be hoped, he will find time to prepare for publication; as such opportunities but rarely occur to engineers so able to profit by them.

16. In the year 1849, the Manufacturing Companies at Lowell purchased of Mr. Boyden, the right to use all his improvements relating to turbines and other hydraulic motors. Since that time it has devolved upon the author, as the chief engineer of those companies, to design and superintend the construction of such turbines as might be wanted for their manufactories, and to aid him in this important undertaking, Mr. Boyden has communicated to him copies of many of his designs for turbines, together

with the results of experiments upon a portion of them; he has communicated, however, but little theoretical information, and the author has been guided, principally, by a comparison of the most successful designs, and such light as he could obtain from writers on this intricate subject.

17. The first designs, prepared by the author, after the arrangement with Mr. Boyden was entered into, were for four turbines of essentially the same dimensions, namely, two for the Suffolk Manufacturing Company, and two for the Tremont Mills, for the purpose of furnishing power for the cotton-mills of these companies at Lowell These turbines were constructed at the Lowell Machine Shop, and were completed in January, 1851.

For the purpose, principally, of estimating the success of these turbines, one of them was fitted up with a complete apparatus for measuring its power, and gauging the quantity of water discharged; the gauging apparatus was afterwards used to make the experiments on the discharge of water over weirs of different proportions, for the purpose of determining, practically, some of the relations required to be known, in order to compute the flow of water through such apertures.

DESCRIPTION OF THE TURBINE

18. The water is conducted from the principal feeder to the mills at Lowell, called the Northern Canal, by an arched canal, or penstock, about ninety feet in length The forebay, inside the wheel-house, is constructed of masonry, and has a general width of twenty feet, and a depth of water of fourteen feet; the channels through which the water passes, are so capacious, that the loss of fall in passing from the Northern Canal to the forebay, is scarcely sensible During the experiments, however, the head of the penstock was partially closed by gates, so that there was a sensible fall at that time.

The entrance of the arched canal is protected by a coarse rack, or grating, for the purpose of preventing large floating substances from entering the forebay; each turbine is also separately guarded by a fine rack, placed in the forebay, which prevents the entrance into the turbine of all floating substances that might be injurious Both racks are made of large extent, to avoid sensible loss of head to the water in passing through them.

The extreme rigor of the New England winter renders it necessary to afford to water-wheels of all descriptions, complete protection from the cold. The result is, that less interruption from frost is experienced, than in many milder climates The wheel-house, in which these turbines are placed, is a substantial brick building, well warmed in the winter by steam.

After passing the turbines, the water is conducted by an arched canal, or raceway, about nine hundred feet in length, to the lower level of the Western Canal, which serves as a feeder to the Mills of the Lawrence Manufacturing Company.

19. Plate I. is a vertical section through the centre of the turbine, and the axis of the supply pipe.

Plate II. is a plan of the turbine, and wheelpit.

Plate III., Figure 1, is a plan of nearly one fourth part of the disc and wheel. Figure 2 is a plan of the whole wheel, the guides, and garniture. Figure 3 is a vertical section through both crowns of the wheel.

The same letters indicate the same parts, in all these three plates.

20. *A, the forebay*, in which the level of the water is nearly the same as in the Northern Canal; it is represented at the usual working height.

21. *B, the surface of the water in the wheelpit*, represented at the lowest height at which the turbine is intended to operate.

22. *C, the masonry of the wheelpit.* The faces towards the wheel, are of granite ashlar work, in blocks containing, generally, from ten to forty cubic feet. The backing is of hard mica slate. The capping course, shown particularly on Plate II., is neatly dressed on its upper surface The whole is compactly laid in hydraulic cement.

23. *D, the floor of the wheelpit.* This floor sustains the weight of part of the supply pipe, and of part of the water in it, and all the rest of the apparatus, excepting the wheel itself and the vertical shaft, which are supported by beams and braces, directly from the side walls of the wheelpit. It was necessary that the floor should have sufficient stiffness to resist the great upward pressure which takes place when the wheelpit is kept dry by pumps, in order to permit repairs to be made. The walls of the wheelpit are built upon the floor;—there was, consequently, no danger of the whole floor being pressed upwards, but the great width of the pit, (twenty-four feet,) would allow the floor to yield in the centre, unless it had great stiffness.

To meet these requirements, three cast-iron beams are placed across the pit, the ends extending about a foot under the walls, on each side; on these are laid thick planks which are firmly secured to the cast-iron beams, by bolts. To protect the thick planking from being worn out by the constant action of the water, they are covered with a flooring of one inch boards, which can be easily renewed when necessary.

24. *E, the wrought iron supply pipe.* This is constructed of plate iron, [illegible] inch thick, riveted together in a similar manner to steam boilers. The horizontal part is nine feet in diameter, the curved part gradually diminishes in diameter, to its junction with the upper curb. The upper end of the supply pipe is terminated by a cast-iron ring *F*, turned smooth on the face, to receive the wooden head gate. The supply pipe is also furnished with the man hole and ventilating pipe *G*, and the leak box *H*. The use of

the latter is, to catch the leakage of the head gate, whenever it is closed for repairs of the wheel; at such times, the leakage is carried off into the raceway, below the wheelpit, by a six inch pipe, furnished with a valve which can be opened and shut at pleasure.

25. *I, the cast-iron curbs.* These conduct the water from the wrought iron supply pipe, to the disc *K.* The curbs are made in four parts, for the convenience of the founder. The surfaces at which they are joined, are turned true in a lathe, packed with red lead, and bolted together with bolts one and a half inches diameter, placed about six inches apart. The general thickness of the iron is one and a quarter inches The flanges are two inches thick. The upper curb has a projection cast on it, to receive the disc pipe. The lower curb is finished on all sides; the outside, to permit the regulating gate to be moved up and down easily; the inside, to present a smooth surface to the water, and to match accurately with the garniture *L.*

The curbs are supported from the wheelpit floor by four columns, two of which are shown at *N N,* plate I., resting on the cast-iron beam *O*; this is placed on the floor, for the purpose of distributing the weight. The centres of the columns are thirteen inches from the outside circumference of the wheel. The beams *N′* rest immediately upon the columns, and the curb upon the beams, the latter projecting over the columns far enough for that purpose. The beams *N′* also act as braces from the wheelpit wall to the curb, and are strongly bolted at each end.

26. *K, the disc.* This is of cast-iron, one and a half inches thick, and is turned smooth on the upper surface, and also on its circumference. It is suspended from the upper curb, by means of the disc pipes *M M.* The disc carries on its upper surface thirty-three guides, or leading curves, for the purpose of giving the water, entering the wheel, proper directions They are made of Russian plate iron, one tenth of an inch in thickness, secured to the disc by tenons, passing through corresponding mortices, cut through the disc, and are riveted on the under-side. The upper corners of the guides, near the wheel, are connected by the garniture *L,* which is intended to diminish the contraction of the streams entering the wheel, when the regulating gate is fully raised. The garniture is composed of thirty-three pieces of cast-iron, or one to fill each space between the guides; these pieces of cast-iron are, necessarily, of irregular form; for a top view of them see *L,* plate III., figure 2. They are also shown in section at plate I. They are carefully fitted to fill the spaces between the guides; above the top of the guides, the adjoining pieces are in contact; they are strongly riveted to the guides, and to each other After they were all fitted and riveted, the disc was put in a lathe, and the top, the periphery, and a part of the inside of the garniture, were turned off, so that it would fit accu-

rately, but easily, to the corresponding part of the lower curb. The disc is not fastened to the lower curb, but is retained in its place, horizontally, by the latter.

27. *M M, the disc pipe.* The disc is fastened to the bottom of the disc pipe by fifteen tap screws, one and a quarter inches in diameter. As there is a vertical pressure on the disc, due to the pressure of the whole head, on its horizontal area, the disc pipe and its fastenings require to be very strong. The pipe is eight and a half inches diameter, inside, or one and a half inches larger than the shaft passing through it, and is one and a quarter inches thick. The upper flange is furnished with adjusting screws, by which the weight is supported upon the upper curb, and which afford the means of adjusting the height of the disc. The escape of water between the upper curb and the upper flange of the disc pipe, is prevented by a band of leather on the outside, which is retained in its place by the wrought iron ring *P*. This ring is made in two segments. The top of the disc pipe, just below the upper flange, has two projections, or wings, which fit into corresponding recesses in the top of the curb; these are to prevent the disc from rotating in the opposite direction to the wheel, to which there is a powerful tendency, arising from the reaction of the water issuing from the guides.

28. *R R, the regulating gate.* This is represented on the section, at plate I., as fully raised, and in this position the wheel would be giving its full power. The gate is of cast-iron, the cylindrical part is one inch thick, the upper part of the cylinder is stiffened by a rib, to which are attached three brackets, one of which is shown at *S*, plate I, and the two others at *S S*, plate II. To these brackets are attached wrought iron rods, by which the gate is raised and lowered. The brackets are attached to the gate at equal distances, and therefore the rods support equal parts of its weight. To one of the rods is attached the rack *V*. The other two rods are attached, by means of links, to the levers *T T*, plate II. The other ends of these levers carry geared arch heads, into which, and into the rack *V*, work three pinions, *W*, of equal pitch and size, fastened to the same shaft. As the fulcrums of the levers *T T*, plate II., are exactly in the middle, between the pitch lines of the arch heads and the points to which the rods are attached, it will be seen, that by the revolution of the pinion shaft, the gate must be moved up or down, equally on all sides. The shaft on which the pinions are fastened, is driven by the worm wheel *X*, plates I. and II.; this is driven by the worm *a*, either by the governor *Y*, or the hand wheel *Z*. The shaft on which the worm *a* is fastened, is furnished with movable couplings, which, when the speed gate is at any intermediate points between its highest and lowest positions, are retained in place by spiral springs; in either of the extreme positions, the couplings are separated by means of a lever, moved by pins in the rack *V*; by this means both the

regulator and hand wheel are prevented from moving the gate in one direction, when the gate has attained either extreme position. If, however, the regulator or hand wheel should be moved in the opposite direction, the couplings would catch, and the gate would be moved. The weight of the gate is counterbalanced by weights attached to the levers *T T*, and by the intervention of a lever to the rack *V*; by this arrangement, both the governor and hand wheel are required to operate, with only the force necessary to overcome the friction of the apparatus.

20. *b b The wheel.* This consists of a central plate of cast-iron, and of two crowns, *c c*, of the same material, to which the buckets are attached. The central plate and the crowns are turned accurately in a lathe, for the purpose of balancing them, and also to diminish, as much as possible, the resistances in moving rapidly through the water. The lower crown is fastened to the central plate, as shown at figures 1 and 8, plate III. These figures also show, at *c c*, the form of the crowns; the upper and lower crowns are precisely alike; they are nine and a half inches wide. At the inner edge, and at the circumference, the thickness is 0.625 inches, and at 5.5 inches from the inner edge, where they have the greatest thickness, they are one inch thick.

By reference to figure 1, plate III, it will be seen that the buckets do not extend to the circumference of the crowns. In the direction of the radius, the ends of the buckets are 0.25 inches from the circumference. This is for the purpose of permitting the wheel to be handled with less danger of injuring the ends of the floats; as these are filed down to an edge, they would be very likely to be damaged during the construction of the wheel, if they were not guarded by the slight projection of the crowns. This construction also enables the grooves in the crowns to afford more perfect support to the ends of the buckets, and also permits a tenon to be nearly at the extremity of the bucket.

The buckets are forty-four in number, and are of the form represented on plate III., figure 1. They are made of plate iron of excellent quality, imported from Russia for the purpose; they are $\frac{9}{32}$ of an inch in thickness, and are secured to the crowns in the following manner.

The crowns having been first turned to the required form, grooves are cut in them of the exact form of the buckets, to the depth *e e*, figure 8, plate III., this depth is 0.1 inches at the edges and 0.6 inches near the middle. These grooves are cut in a machine contrived for the purpose, in which the cutting tool is guided by a cam. Three mortices for each bucket are then cut through each crown; corresponding tenons are left on the buckets; the latter are bent to the required form, by means of a pair of dies, prepared for the purpose, the plate iron having been first moderately heated. The tenons of all the buckets are then entered into the mortices in both crowns, the latter are then drawn together, by means of [illegible] number of screws applied to different parts of the circumference, and when the edges of the buckets are drawn into the bottom of the grooves, the tenons are riveted on the opposite sides. This construction gives great stability to the buckets, and permits the [illegible] of very thin iron.

30. *d d The vertical shaft.* This is of wrought iron, and is accurately turned in every part.

The diameters are as follows:—

Below the hub of the wheel, ...	7 inches.
In the hub of the wheel, ...	7¼ "
Between the top of the hub and the lower bearing,	7 "
Between the bottom of the lower bearing and the hub of the bevel gear, ...	8 "
In the hub of the bevel gear, ...	8¼ "
From the top of the hub of the bevel gear to the suspension box, ...	8 "

By reference to plate I, it will be seen that the shaft does not run upon [illegible] step at the bottom, but upon a series of collars, resting upon corresponding projections in the suspension box e'. The part of the shaft on which the collars are placed, is made separate from the main shaft, and is joined to it at f, by means of a socket in the top of the main shaft, which receives a corresponding part of the collar piece. The collars are made of cast steel; they [illegible] separately screwed on, and keyed to a wrought iron spindle.

31. *e' The suspension box* This is made in two parts, to admit of its being taken off, and put on the shaft; it is lined with babbit metal, [illegible] soft composition consisting principally of tin It is found that bearings thus lined will carry from fifty to a hundred pounds to the square inch, with every appearance of durability.

32. *$f'f'$, The upper and lower bearings.* These are of cast-iron, lined with babbit metal; they are retained in position, horizontally, by means of adjusting screws; vertically, their weight is sufficient. The parts of the shaft inside the hubs of the wheel and the bevel gear, are made slightly tapering, about $\frac{1}{16}$ of an inch in diameter in the length of the hubs; the hubs [illegible] bored out with the [illegible] taper, but a very little smaller in diameter; they are then drawn on by a powerful screw purchase, and in this manner are made to fit very tight. To prevent danger of bursting the hubs, they are before being drawn on or bored out, strongly hooped with wrought iron hoops, driven on hot.

33. The suspension box e' (art. 31,) rests upon the gimbal g, plates I. and II. The gimbal itself is supported on the frame $h\,h$, by adjusting screws, which give the means

of raising and lowering the suspension box, and, with it, the vertical shaft and wheel. It will be perceived, by the arrangement of the bearings above and below the bevel gear, that no lateral strain can be thrown upon the suspension box. The construction of the shaft will evidently not admit, with safety, of lateral strain at the suspension box, and it is accordingly so arranged that this box is free to oscillate horizontally in any direction, a small quantity, in case any irregularity in the form of the shaft should require it.

The lower end of the shaft is fitted with a cast steel pin *z*, plate I. This is retained in its place by the step, which is made in three parts, and lined with casehardened wrought iron. The step is furnished with adjusting screws, by means of which the shaft can be moved horizontally in any direction, a small distance.

The weight of the wheel, upright shaft, and bevel gear, is supported by means of the suspension box *e,'* on the frame *k*, which rests upon the long beams *m*, reaching across the wheelpit, and supported at the ends by the masonry, and also at intermediate points by the braces *n n*.

From economical considerations *the diffuser*, described at art. 12, was omitted at the Tremont Turbine; a large majority of the turbines in use at Lowell, however, are fitted up with that apparatus.

84. The following are some of the dimensions of the turbine, carefully taken after the parts were finished:—

Height between the upper and lower crowns, at the outer extremities of the buckets, a mean of 44 measurements, ...	0.9314	foot.
Height between the upper and lower crowns, at the inner extremities of the buckets, a mean of 44 measurements, ...	0.9368	“
Height between the crowns, at a point 5.5 inches from the inner edges of the crowns, (designed to be 0.75 inches less than at the inner edges,)	0.8743	“
Shortest distance between the outer extremities of the buckets and the next adjacent buckets, a mean of 132 measurements,	0.18757	“
Shortest horizontal distance between two adjacent guides, taken at the top of the circumferential part of the disc, a mean of 33 measurements, ...	0.1960	“
Do. do. at the bottom of the garniture, ...	0.2117	“
Do. do. half-way up between the disc and the garniture, . .	0.2044	“
The shortest distance between the guides, by a mean of the whole 99 measurements, ...	0.20403	“
Height from the top of the circumferential part of the disc to the bottom of the garniture, a mean of 33 measurements, . .	0.97090	“

35. The following are some of the most important dimensions of the apparatus; they are taken from the original designs, which were very closely followed in the construction.

Diameter of the exterior circumference of the crowns of the wheel,	8.333	feet.
" " outer extremities of the buckets,	8.292	"
" " interior edges of the crowns, and inner edges of the buckets,	6.750	"
" " outside of the cylindrical part of the regulating gate,	6.729	"
" " inside of the cylindrical part of the regulating gate,	6.562	"
" " of the outside of the lower curb, taken below the flange,	6.512	"
" " inside of the lower curb, taken at the top,	6.333	"
" " inside of the lower curb, taken at the top of the guides,	6.167	"
" " lower part of the disc,	6.729	"

DESCRIPTION OF THE APPARATUS USED IN THE EXPERIMENTS ON THE TREMONT TURBINE.

36. The details of this apparatus are represented on plate IV.

The useful effect was measured with a Prony dynamometer, represented in sectional elevation at figure 1, and in plan at figure 2.

37. *The friction pulley A* is of cast-iron 5.5 feet in diameter, two feet wide on the face, and three inches thick. It is attached to the vertical shaft by the spider *B*, the hub of which occupies the place on the shaft intended for the bevel gear.

The friction pulley has, cast on its interior circumference, six lugs, *C C*, corresponding to the six arms of the spider. The bolt holes in the ends of the arms are slightly elongated in the direction of the radius, for the purpose of allowing the friction pulley to expand a little as it becomes heated, without throwing much strain upon the spider. When the spider and friction pulley are at the same temperature, the ends of the arms are in contact with the friction pulley. The friction pulley was made of great thickness for two reasons. When the pulley is heated, the arms cease to be in contact with the interior circumference of the pulley, consequently they would not prevent the pressure of the brake from altering the form of the pulley. This renders great stiffness necessary in the pulley itself. Again, it is found that a heavy friction pulley insures more regularity in the motion, operating, in fact, [illegible] [illegible] fly-wheel, in equalizing small irregularities.

38. *The brakes E and F* are of maple wood; the two parts are drawn together by the wrought iron bolts *G G*, which are two inches square.

39. *The bell crank H* carries at one end the scale *I*, and at the other the piston of the hydraulic regulator *K*; this end carries also the pointer *L*, which indicates the level of the horizontal arm. The vertical arm is connected with the brake *F*, by the link *M*, figure 3.

40. *The hydraulic regulator K*, figures 1, 2, and 5, is a very important addition to the Prony dynamometer, first suggested to the author by Mr. Boyden in 1844. Its office is to control and modify the violent shocks and irregularities, which usually occur in the action of this valuable instrument, and are the cause of some uncertainty in its indications.

The hydraulic regulator used in these experiments, consisted of the cast-iron cylinder *K*, about 1.5 feet in diameter, with a bottom of plank, which was strongly bolted to the capping stone of the wheelpit, as represented in figure 1. In this cylinder, moves the piston *N*, formed of plate iron 0.5 inches thick, which is connected with the horizontal arm of the bell crank by the piston rod *O*. The circumference of the piston is rounded off, and its diameter is about $\frac{1}{16}$ inch less than the diameter of the interior of the cylinder. The action of the hydraulic regulator is as follows. The cylinder should be nearly filled with water, or other heavy inelastic fluid. In case of any irregularity in the force of the wheel, or in the friction of the brake, the tendency will be, either to raise or lower the weight; in either case the weight cannot move, except with a corresponding movement of the piston. In consequence of the inelasticity of the fluid, the piston can move only by the displacement of a portion of the fluid, which must evidently pass between the edge of the piston and the cylinder, and the area of this space being very small, compared to the area of the piston, the motion of the latter must be slow; giving time to alter the tension of the brake screws before the piston has moved far. It is plain that this arrangement must arrest all violent shocks, but, however violent and irregular they may be, it is evident that, if the mean force of them is greater in one direction than in the other, the piston must move in the direction of the preponderating force, the resistance to a slow movement being very slight. A small portion of the useful effect of the turbine must be expended in this instrument; probably less, however, than in the rude shocks the brake would be subject to without its use.

41. For the purpose of ascertaining the velocity of the wheel, a counter was attached to the top of the vertical shaft, so arranged that a bell was struck at the end of every fifty revolutions of the wheel.

42. To lubricate the friction pulley, and at the same time to keep it cool, water was let on to its surface in four jets, two of which are shown in figure 2, plate IV.

These jets were supplied from a large cistern, in the attic of the neighboring cotton-mill, kept full, during the working hours of the mill, by force-pumps. The quantity of water discharged by the four jets was, by a mean of two trials, 0.0288 cubic feet per second.

In many of the experiments with heavy weights, and consequently slow velocities, oil was used to lubricate the brake, the water, during the experiment, being shut off. It is found that, with a small quantity of oil, the friction between the brake and the pulley, is much greater than when the usual quantity of water is applied; consequently, the requisite tension of the brake screws is much less with the oil, as a lubricator, than with water. This may not be the whole cause of the phenomenon, but, whatever it may be, the ease of regulating in slow velocities is incomparably greater with oil as a lubricator, than with water applied in a quantity sufficient to keep the pulley cool. The oil was allowed to flow on in two fine continuous streams;—it did not, however, prevent the pulley from becoming heated sufficiently to decompose the oil, after running some time, which was distinctly indicated by the smoke and peculiar odor. When these indications became very apparent, the experiment was stopped, and water let on by the jets, until the pulley was cooled. As the pulley became heated, the brake screws required to be gradually slackened.

In the experiments, in table II., the lubricating fluid was as follows.

In the first twenty-six experiments, water alone was used.

In the four experiments numbered from 27 to 30, three gallons of linseed oil were used.

In all the experiments requiring a lubricator, and numbered from 31, to 48, inclusive, linseed oil was used.

In experiments 49 and 50, resin oil was used.

In experiments numbered from 51 to 60, inclusive, water alone was used.

In experiment 61, resin oil was used.

In experiment 62, resin oil and a small stream of water were used;—in the latter part of the experiment, a good deal of steam was generated by the heat of the friction pulley.

In experiment 63, resin oil alone was used.

In experiments numbered from 66 to 72, inclusive, water alone was used.

In experiments numbered from 73 to 79, inclusive, resin oil and a small stream of water were used.

In experiments numbered from 81 to 84, inclusive, water alone was used.

In experiments 85 and 86, resin oil and a small stream of water were used.

In experiment 87, resin oil alone was used.

In experiments 90 and 91, water alone was used.

In experiment 92, resin oil and a small stream of water were used.

43. A special apparatus was provided to indicate the direction in which the water left the wheel. For this purpose the vane *P*, figures 1, 6, and 7, plate IV, was placed near the circumference of the wheel, and was keyed on to the vertical shaft *Q*, which turned freely on a step resting on the wheelpit floor. The upper end of the shaft carried the hand *R*, figures 1 and 4, and directly under the hand was placed the graduated semicircle *S*, divided into 180°. When the vane was parallel to a tangent to the circumference of the wheel, drawn through the point nearest to the axis of the vane, and the vane was in the direction of the motion of the wheel, the hand pointed at 0°, and, consequently, when the vane was in the direction of the radius of the wheel, the hand pointed at 90°. To prevent sudden vibrations of the vane, a modification of the hydraulic regulator was attached to the lower part of the vane shaft. This apparatus is represented in detail by figures 6 and 8.

44. The quantity of water discharged by the wheel was gauged at a weir erected for the purpose at the mouth of the wheelpit. It is represented on plate V.

Figure 1 is a plan, and figure 2 a section, showing the relative positions of the turbine *A*, the grating *B*, the gauge box *C*, and the two divisions or bays of the weir, *D*, and *E*.

As the water issued from the orifices of the turbine with considerable force, particularly when the velocity of the wheel was much quicker or slower than that corresponding to the maximum coefficient of effect, there were often such violent commotions in the wheelpit, that, unless some mode was adopted to diminish them before the water reached the weir, or even the place where the depths on the weir were measured, it would have been impossible to make a satisfactory gauge of the water. For this purpose the grating *B*, figures 1 and 2, was placed across the wheelpit. This grating presented numerous apertures, nearly uniformly distributed over its entire area, through which the water must pass. In the experiments with a full gate, the fall from the upper to the lower side of the grating was generally from three to four inches. The combined effect of this fall and of the numerous small apertures, was, to obliterate almost entirely the whirls and commotions of the water above the grating. About 4.5 feet in length of the grating between *F* and *G*, figure 1, was so nearly closed, that but little water passed through that part of the grating;—this made it very quiet in the vicinity of the gauge box *C*.

Figure 3, plate V., is an elevation of the weir. The two bays *D* and *E* were of nearly equal length,—the crest of the weir was almost exactly horizontal, and the extreme variation did not exceed 0.01 inch. The crest of the weir was of cast-iron, planed on the upper edge *H*, figures 2 and 4, and also on the upstream face, to a point 1.125 inches below the top;—below this, at *I*, figure 4, there was a small bevel, also planed, the slope of which, on an average, was $\frac{3}{16}$ inch in a height of $\frac{5}{8}$ inch; the remainder of the casting was unplaned. The crest of the weir *H* was $\frac{1}{4}$ inch thick, and was horizontal. The upstream edge, at *H*, was a sharp corner. The ends of the weir *K*, figures 1, 2, and 3, were of wood, and of the same form as the crest *H*, except that there was no bevelled part corresponding to *I*, figure 4. The crest of the weir *H* was about 6.5 feet above the floor of the wheelpit. The ends of the weir *K* projected from the walls of the wheelpit, and also from the central pier, a mean distance of 1.235 feet. The length of the bay *D*, was 8.480 feet, and of the bay *E*, 8.491 feet, making the total length of the weir 16.98 feet.

45. The depth of the water on the weir was taken in the gauge box *C*, figures 1 and 2, plate V., by means of the hook gauge *L*, which is represented in detail by figures 9, 10, and 11, plate IV.

The hook gauge is the invention of Mr. Boyden,* and is an instrument of inestimable value in hydraulic experiments. All other known methods of measuring the heights of the surface of still water, are seriously incommoded by the effects of capillary attraction; this instrument, on the contrary, owes its extraordinary precision to that phenomenon. At figure 10, plate IV., the point of the hook *A*, is represented as coinciding with the surface of the water. If the point of the hook should be a very little above the surface, the water in the immediate vicinity of the hook, would, by capillary attraction, be elevated with it, causing a distortion in the reflection of the light from the surface of the water. The most convenient method of observing with this instrument, according to the experience of the author, is, first, to lower the point of the hook, by means of the screw, to a little distance below the surface; — then to raise it again slowly by the same means, until the distortion of the reflection begins to show itself, — then to make a slight movement of the screw in the opposite direction, so [illegible] just to cause the distortion to disappear; the point will then be almost exactly at the level of the surface.

With no particular arrangements for directing light on the surface, differences in height of 0.001 feet are very distinct quantities; but by special arrangements for light and vision, differences of 0.0001 feet might be easily appreciated.

As this instrument cannot be efficiently used in a current, it was placed in the box *C*, in which the communication with the exterior was maintained by the hole *M*, when, by partially obstructing this communication, the extent of the oscillations could be diminished at will.

---

* In *Versuche über den ausfluss des wassers durch schieber, hähne, klappen und ventile, by Julius Weisbach, Leipzig*, 1842, page 1, is described [illegible] instrument for observing heights of water, having a slight resemblance to the hook gauge; it was however used by Boyden in [illegible] more perfect form, several years previous to the publication of that work.

For the most perfect observations, it is essential that the surface of the water should be at rest. If, however, it should oscillate a little, a good mean may be obtained by adjusting the point of the hook to a height at which it will be visible above the surface of the water only half the time.

The movable rod to which the hook was attached, was of copper, and graduated to hundredths of foot, but, by means of the vernier, thousandths were measured, and in some cases ten thousandths were estimated. In later, and more perfect forms of this instrument, the point of the hook is immediately under the graduation.

46. The heights of the water in the forebay, and in the wheelpit, were taken by means of gauges, placed in the gauge boxes *p* and *q*, plate II. These boxes were similar to the box *C*, plate V., in which the hook gauge was placed. Both gauges were graduated to feet and hundredths, and both had the same zero point, viz., the level of the crest of the weir, [illegible] that the difference in the readings at the two gauges, gives, at once, the fall acting upon the wheel; and the difference between the depths of the water on the weir, as observed at the hook gauge, and the reading at the gauge *q*, gives the fall at the grating.

In consequence of want of space in plate II., the gauge box *p* is not represented in its true position,—it was actually in front of the head gate, and about six feet distant.

47. The heights of the regulating gate were taken at the rack *V*, plate I. The weights used for measuring the useful effect, were pieces of pig-iron of various sizes, each of which had been distinctly marked with its weight by Mr. O. A. Richardson, the official sealer of weights and measures for the City of Lowell.

MODE OF CONDUCTING THE EXPERIMENTS

48. A separate observer was appointed to note each class of data; the time of each observation was also noted, which gave the means of identifying simultaneous observations. To accomplish this, each observer was furnished with a watch having a second hand;—the watch by which the speed of the wheel was observed, was taken as the standard; all the others were frequently compared with it, and when the variations exceeded ten or fifteen seconds, they were either adjusted to the standard, or the difference noted.

This mode of observing must, evidently, lead to more precise results than that in which [illegible] single observer, however skilful, undertakes to note all the phenomena, or even several of them. By the method adopted, a regular record is made of the state of things at very short intervals, furnishing the data for a mean result for any required period, and also the means of detecting, in most cases, the causes of apparent discrepancies. It also relieves the experimenter from the distraction of having numerous exact observations to make in a very short time, and leaves him much more at liberty to exercise a vigilant watch over the general course of the experiment.

49. As it may be useful to experimenters, not accustomed to this mode of observing, and, at the same time, afford the reader some means of judging of the accuracy of the results obtained in these experiments, the following extracts are given from the original note-books. The extracts include the data observed for experiment numbered 30 in table II. This experiment is selected, simply, because it gave the maximum coefficient of effect.

WEIGHT IN THE SCALE

Extract from the note-book of the author, who superintended the experiments.

	1,498 lbs. 10½ oz.
4h, 48′, added	26 " 0¼ "
Weight for the next experiment, . .	1,524 lbs. 10¾ oz.

SPEED OF THE WHEEL

Extract from the note-book of Mr. Charles Leonard.

Times at which the bell struck.			Differences	Times at which the bell struck			Differences	Times at which the bell struck.			Differences
H.	m.	sec.	Seconds.	H.	m.	sec.	Seconds.	H.	m.	sec.	Seconds.
4.	55.	58.00		5	0	52.00	59.00	5.	4.	47.00	59.00
	56.	56.50	58.50		1.	50.75	58.75		5	45.50	58.50
	57.	55.25	58.75		2.	49.50	58.75		6.	44.25	58.75
	58.	54.25	59.00		3.	48.00	58.50		7.	43.00	58.75
	59.	53.00	58.75								

NOTE. The bell struck once in every fifty revolutions of the wheel.

ELEVATION OF THE POINTER ON THE BELL CRANK

Time.			Height of pointer, in feet.	Time			Height of pointer, in feet	Time			Height of pointer, in feet
H.	min.	sec.		H.	min.	sec.		H.	min.	sec.	
4.	55.	0.	0.19	4.	59.	30.	0.20	5.	4.	0.	0.17
		30.	0.13	5.	0.	0.	0.18			30.	0.18
	56.		0.18			30.	0.19		5.		0.24
		30.	0.14		1.		0.21			30	0.18
	57.		0.15			30.	0.17		6.		0.19
		30	0.19		2.		0.20			30.	0.19
	58.		0.20			30.	0.19		7.		0.16
		30.	0.19		3.		0.19			30.	0.14
	59.		0.21			30.	0.19				

NOTE. The extremity of the pointer was 6.5 feet from the fulcrum of the bell crank. When the horizontal arms of the bell crank were level, the height of the pointer was 0.20 feet.

HEIGHT OF THE WATER ABOVE THE WHEEL

Taken in the forebay by Mr. John Newell.

Time			Height, in feet	Time			Height in feet	Time			Height, in feet.
H.	min.	sec.		H.	min.	sec.		H.	min.	sec.	
4.	55.	0.	15.100	4.	59.	30.	15.110	5.	4.	0.	15.120
		30.	15.100	[illegible]	0.		15.115			30.	15.120
	56.		15.100			30.	15.120		5.		15.120
		30.	15.100		1.		15.120			30.	15.115
	57.		15.110			30.	15.110		6.		15.115
		30.	15.115		2.		15.105			30.	15.110
	58.		15.110			30.	15.100		7.		15.110
		30.	15.100		3.		15.115			30.	15.110
	59.		15.105			30.	15.125				

NOTE. The top of the weir is the zero point of the gauge in the forebay.

HEIGHT OF THE WATER AFTER PASSING THE WHEEL

Taken in the wheelpit by Mr. Lloyd Hixon.

Time.			Height in feet.	Time.			Height in feet.	Time.			Height, in fe[illegible]
H.	min.	sec.		H.	min.	sec.		H.	min.	sec.	
4.	56.	0.	2.20	5.	0.	0.	2.21	5.	4.	0.	2.22
		30.	2.21			30.	2.21			30.	2.21
	57.		2.21		1.		2.21		5.		2.21
		30	2.21			30.	2.21			30.	2.21
	58.		2.21		2.		2.21		6.		2.21
		30	2.21			30	2.21			30.	2.20
	59.		2.20		3.		2.20		7.		2.22
		30.	2.21			30.	2.20			30.	2.20

NOTE. The top of the weir is the zero point of the gauge in the wheelpit.

HEIGHTS OF THE WATER ABOVE THE WEIR BY THE HOOK GAUGE.

Observed by Mr. David Huffaly.

Time			Height, in feet	Time			Height, in feet	Time			Height, in feet
H	min	sec.		H	min	sec		H.	m	sec	
4.	57.	5.	1.8710	5.	1.	10.	1.8690	5.	4.	35.	1.8730
	58.	15.	1.8710		1.	45.	1.8700		5.	50.	1.8725
	58.	50.	1.8720		2.	15.	1.8720		6.	25.	1.8725
	59.	20.	1.8730		2.	50.	1.8720		6.	55.	1.8725
	59	50.	1.8715		3.	15.	1.8715		7.	20.	1.8720
5	0.	15.	1.8715		3.	40.	1.8715		7.	45.	1.8715
	0.	45.	1.8705		4.	5.	1.8730				

NOTE. The zero of the hook gauge was 0.002 foot *below* the top of the weir.

DIRECTION OF THE WATER LEAVING THE WHEEL.

Observed at the [redacted] index by Mr. John C. Woodward.

Time			Direction.		Time			Direction.		Time.			Direction.	
H	m	sec.	deg	min.	H	min	sec.	deg.	min.	H.	min.	sec.	deg.	min.
4.	57.	0.	59.	0.	5.	1.	0.	57.	0.	5.	5.	0.	58.	0.
		30.	57.	0.			30.	59.	30.			30.	59.	30.
	58.		59.	0.		2.		58.	0.		6.		59.	30.
		30.	58.	0			30.	57.	0.			30.	57.	0.
	59.		58	0.		3.		60.	0.		7.		59.	0.
		30.	58.	30.			30.	58.	0.			30.	57.	30.
5.	0.		57.	0.		4.		59.	0.		8.		59.	0.
		30.	57.	30.			30.	60	0.					

NOTE. When the vane pointed in the direction of the radius of the wheel, the reading of the index was 90°. 0° was in the direction of the motion of the wheel.

50. Previously to the commencement of the experiments, the apparatus for measuring the useful effect was carefully adjusted. The bell crank was balanced when there were no weights in the scale. For this purpose the link *M*, figure 3, plate IV., was removed, and the chamber of the hydraulic regulator filled with water; — weights were then applied to the top of the bell crank, near the end to which the hydraulic regulator was attached, until the whole was in equilibrium; — the final adjustment was made, by placing a weight of about two pounds at the extremity of one of the horizontal arms of the bell crank, — the arm was retained horizontally until a signal was given, when it was left at liberty to descend, and the time occupied in descending [redacted] certain distance was noted; — the weight was then removed to the extremity of the other arm, and the [redacted] process repeated. The balance weights were altered until the times of descent

were equal. To overcome, as much as possible, the friction of the fulcrum, the pin forming it was lubricated with sperm oil, and, during the descent, the head of the pin was struck lightly and rapidly with a small hammer.

After the bell crank was satisfactorily balanced, the link *M* was reattached, and the brake adjusted, by means of the screw which formed the connection between the link and the brake. It was adjusted so that a line upon the brake was perpendicular to the axis of the link, when the horizontal arm of the bell crank was horizontal. The length of the brake was then measured upon this line.

The length of the brake as thus measured was found to be . . 9.745 feet.
The effective length of the vertical arm of the bell crank was 4.500 "
And the effective length of the horizontal arm to which the scale was hung, was . . . . 5.000 "
Consequently, the effective length of the brake was $\frac{9.745 \times 5}{4.5} = 10.827778$ "

51. The gauges in the forebay, and in the wheelpit, were carefully adjusted by levelling from the top of the weir. This was repeated by different persons, so as to remove all chance of error.

52. The hook gauge was compared with the weir, by a different method. When the regulating gate of the turbine was shut down as tight as possible, it was still found that a quantity of water leaked into the wheelpit, exceeding, a little, the quantity that leaked out of the wheelpit, so that a small quantity continued to run over the weir. The principal leak into the wheelpit was between the regulating gate and the lower curb, the leather packing not being perfectly adjusted. The hook gauge was firmly attached to a post, placed in the wheelpit for that purpose, and at a height known to be nearly correct. The regulating gate was closed, and after the water had arrived at a uniform state, the height of the water at the hook gauge was noted, and, at the same time, the depths of the water on the weir were measured directly with a graduated rule. To perform this accurately, a board, about four inches long, was held by an assistant on the crest of the weir, at the place where it was intended to measure the depth; — the author then applied the rule, previously well dried, vertically, on the top of the weir, in front of the board. On first immersing the rule, the water in contact with it did not stand at the true level of the surface, but formed a little hollow around the rule; it immediately commenced rising, however, and after a few moments came to a level, which was indicated by the reflection of a light from the surface, a lamp being held by an assistant, in a proper position, for that purpose.

The depths on the weir, taken in the manner just described, February 20th, 1851, were as follows:

	Depths on the westerly bay of the weir Inches	Depths on the easterly bay of the weir Inches
	0.37	0.36
	0.36	0.36
	0.37	0.36
	0.37	0.36
Means . .	0.3675	. . . . 0.36
Or in feet .	0.0306	. . . . 0.0300

While the heights given in the preceding table were being measured, the depth by the hook gauge was constantly 0.0318 foot; consequently, by this comparison, the zero of the hook gauge was 0.0012 foot below the mean height of the top of the weir, in the westerly bay, and 0.0018 foot below the mean height in the easterly bay, or 0.0015 foot below the mean height in both bays. A similar comparison was made February 22d, 1851, when the zero of the hook gauge was found to be 0.0024 foot below the mean height of the weir. The mean of the two comparisons, or 0.0020 was adopted as the correction to be subtracted from the reading of the hook gauge, to give the mean depth upon the weir.

58. During the experiments, the levels of the water in the upper and lower canals, were maintained nearly uniform. The height of the lower canal, at the place where the water, passing the weir, fell into it, varied a little, depending upon the quantity of water discharged by the wheel. It was highest when the wheel was running with the regulating gate fully raised, and the brake removed; under these circumstances the surface of the water was from 0.3 foot to 0.4 foot below the top of the weir. In the other experiments with the regulating gate fully raised, the fall from the top of the weir to the surface of the water in the lower canal, was from 0.4 foot to 0.6 foot. The brackets *N* and the planks *O*, figure 2, plate V., were not put on until after the turbine experiments were concluded, so that the water passing the weir, met with no obstruction until it struck the water in the lower canal.

It will be seen by the experiments on the weir, (art. 127,) that the obstruction, caused by the planks, was scarcely appreciable, which renders it certain that the effect of the lower canal, in obstructing the flow over the weir, must have been entirely inappreciable.

DESCRIPTION OF TABLE II, CONTAINING THE EXPERIMENTS UPON THE TURBINE AT THE TREMONT MILLS.

54. The data obtained by direct observation, and the results deduced from them by calculation, are arranged together, for convenience of reference, in table II.

The columns numbered 1, 2, and 3, require no further explanations than are contained in the several headings.

55. Column 4. *Height of the regulating gate.* The three first experiments were made under circumstances identical in every thing, except that the height of the regulating gate was varied a little, for the purpose of ascertaining the height giving the maximum coefficient of effect. The mean height between the crowns of the wheel, at the inner edges of the buckets, was 0.9368 feet, or 11.2416 inches; the curvature of the disc and garniture, however, rendered it necessary to raise the gate rather more than this, in order to present the most favorable aperture. By a comparison of the first three experiments, it appears that the most favorable result was obtained, with the gate raised to a height of 11.50 inches, or a little less; the succeeding experiments, numbered from 4 to 50, inclusive, were made with the regulating gate raised to the full height, or to 11.50 inches, nearly. A comparison of the first three experiments will show that there could be no appreciable difference in the results, that could be attributed to the small differences in the heights of the gate, in the experiments numbered from 4 to 50, inclusive, and they are accordingly all classed together, as experiments with a full gate, the small difference in the heights being accidental.

The experiments numbered from 51 to 64, inclusive, were made with the gate raised 8.55 inches, or three-fourths of the full height, nearly. Those numbered from 65 to 76, inclusive, were made with the gate at very nearly half of the full height. Those numbered from 77 to 79, inclusive, were made with the gate at about seven eighths of the full height. Those numbered from 80 to 89, inclusive, were made with the gate at about one fourth of its full height. And the last three experiments were made with the gate raised one inch.

56. Column 5. *Time.* The times entered under the heads *beginning, and ending, of the experiments,* are taken from the notes of the "speed of the wheel," and indicate the times at which the bell, attached to the counter, was struck, which, by a comparison of the various note-books, appeared, by the regularity of the observations, to be the most suitable for the commencement and termination of the experiment.

57. COLUMN 6 *Duration of the experiment*, is obtained by taking the differences of the times of the beginning, and ending of the experiment, as given in column 5.

58. COLUMN 7. *Total number of revolutions of the wheel during the experiment.* This is obtained from the note-book of the "speed of the wheel," by counting the number of observations of the times at which the bell was struck; this number, less one, multiplied by 50, which is the number of revolutions of the wheel to each stroke of the bell, gives the number of revolutions during the experiment.

59. COLUMN 8. *Number of revolutions of the wheel per second*, is obtained by dividing the total number of revolutions of the wheel, by the duration of the experiment

60. COLUMN 9. *The weight in the scale*, requires no explanation.

61. COLUMN 10. *Useful effect, or the friction of the brake, in pounds avoirdupois, raised one foot per second* This is obtained by multiplying together the weight in the scale, and the velocity that the point of application of the weight, tends to take Or, in other words, the product of the weight into the velocity that the weight would actually take, if, for an infinitely short time, the brake, and the apparatus connecting it with the weight, were rigidly connected with the friction pulley.

The effective length of the brake, including the leverage due to the different lengths of the arms of the bell crank, was 10.827778 feet (art. 50). The circumference of a circle of this radius is 68.0329 feet. This circumference is a constant for all the experiments in which any useful effect was produced, and column 10 was obtained by the product of this constant, the weight, and the number of revolutions of the wheel per second. The computation was performed by logarithms, and if the results given in the tables should be verified by actual multiplications, minute differences would, no doubt, be detected.

62. COLUMNS 11 and 12. *Heights of the water in the forebay and in the wheelpit.* These heights are all referred to the top of the weir, consequently, the differences give the fall acting upon the wheel.

63. COLUMN 13. *Total fall acting upon the wheel.* These are the differences referred to in the last sentence. In experiments 27 and 28, observations were taken in the ventilating pipe *G*, plate I, for the purpose of estimating the loss of fall to this part of the supply pipe;—it was not convenient, however, to measure these heights with complete accuracy. In experiment 27, the height of the water in the ventilating pipe was 0.106 feet below the level in the forebay;—in experiment 28 the difference was found to be 0.102 feet;—in experiment 30, which gave the maximum coefficient of effect, the quantity of water discharged by the wheel, was a little less than in either experiment 27, or 28. We may, therefore,

conclude, that when the regulating gate was fully raised, and the wheel running with the velocity giving the maximum coefficient of effect, the fall acting upon the wheel being 12.903 feet, the loss of fall from the forebay to the ventilating pipe, was very nearly 0.10 feet.

64. COLUMN 14. *Depth of water on the weir.* The depths on the weir were observed with the hook gauge, described at art. 45.

65 COLUMN 15. *Quantity of water passing the weir.* These quantities have been calculated by the formula

$$Q = 3.33\,(l - 0.1\,n\,h)\,h^{\frac{3}{2}},$$

in which

Q = Quantity, in cubic feet per second,
l = The total length of the weir, in feet.
n = The number of end contractions in the weir.
h = The depth on the weir, in feet.

It is unnecessary here to discuss the reasons that have induced the author to adopt this formula, [illegible] different from any that has been used heretofore, as the subject is fully considered in another part of this work.

A small quantity of water entered the wheelpit without passing through the wheel; there was also a small quantity that leaked out by passing through the floor of the wheelpit; the latter quantity, when the depth on the weir was 0.496 feet, was estimated at 0.409 cubic feet per second; see art. 130. As these quantities were very minute, and tended to compensate each other, they have been neglected, and the quantity computed as passing the weir is taken for the quantity discharged by the wheel.

66. COLUMN 16. *Total power of the water.* This column is obtained by multiplying together the total fall acting upon the wheel, the quantity of water passing the weir per second, and the weight of [illegible] cubic foot of water. The temperature of the water was constantly at 32° Fahrenheit, it was nearly pure, and the weight of a cubic foot was taken at 62.375 pounds avoirdupois.

The water of the Merrimack River is always remarkably free from impurities, held in solution, flowing, as it does, from, and through a primitive formation, covered with a sterile soil. In midwinter, at which season these experiments were made, it is more than ordinarily pure, as at that season the surface of the country is usually covered with snow, and the soil frozen to a considerable depth, the river itself, wherever it flows with a moderate current, is frozen over, [illegible] that heavy carriages can often pass with safety, and at the time when these experiments were made, the river for about eighteen miles before it reached the turbine, was covered with a solid coating of ice, with scarcely an opening in the whole distance. When the river is thus frozen, the water flows along under the ice, entirely free from floating particles of ice, even in the most severe weather.

As the author had frequently felt the want of a table of the absolute weights of a cubic foot of water at different temperatures, he, several years since, computed the following table.

In the *Encyclopedia Britannica*, seventh edition, vol. 21, page 840, is given the following extract from the British act of Parliament, establishing the standards for weights and measures.

"Provided always, and be it enacted, that in all cases of dispute respecting the correctness of any measure of capacity, arising in a place where recourse cannot conveniently be had to any of the aforesaid verified copies or models of the standard measures of capacity, it shall and may be lawful, to and for, any justice of the peace, or magistrate, having jurisdiction in such place, to ascertain the content of such measure of capacity by direct reference to the weight of pure or rain water which such measure is capable of containing; ten pounds avoirdupois weight of such water, at the temperature of 62° by Fahrenheit's thermometer, being the standard gallon ascertained by this act, the same being in bulk equal to 277.276; 1822 (1828, 277.274) cubic inches, and so in proportion," etc. 277.274 cubic inches was taken, as it appeared to be the latest determination.

In the first volume of the *Traite de Chimie*, by *J. J. Berzelius*, second French edition, Paris, 1846, there is given a table of the specific gravities of pure water, at different temperatures of the centigrade scale, deduced from Haellstroem's experiments.

From these two authorities were derived the data for the following table.

TABLE I.

WEIGHT OF A CUBIC FOOT OF PURE WATER AT DIFFERENT TEMPERATURES.

Temperature, in degrees of Fahrenheit's thermometer	Weight in air of a cubic foot of pure water. Pounds avoirdupois.	Temperature, in degrees of Fahrenheit's thermometer	Weight in air of a cubic foot of pure water. Pounds avoirdupois.	Temperature, in degrees of Fahrenheit's thermometer	Weight in air, of a cubic foot of pure water. Pounds avoirdupois.
32	62.375	50	62.368	69	62.278
33	62.377	51	62.365	70	62.272
34	62.378	52	62.363	71	62.264
35	62.379	53	62.359	72	62.257
36	62.380	54	62.356	73	62.249
37	62.381	55	62.352	74	62.242
38	62.381	56	62.349	75	62.234
39 (max.)	62.382	57	62.345	76	62.225
39.83	62.382	58	62.340	77	62.217
40	62.382	59	62.336	78	62.208
41	62.381	60	62.331	79	62.199
42	62.381	61	62.326	80	62.190
43	62.380	62	62.321	81	62.181
44	62.379	63	62.316	82	62.172
45	62.378	64	62.310	83	62.162
46	62.376	65	62.304	84	62.152
47	62.375	66	62.298	85	62.142
48	62.373	67	62.292	86	62.132
49	62.371	68	62.285		

67. COLUMN 17. *Ratio of the useful effect to the power expended.* This column is obtained by dividing the numbers in column 10 by those in column 16.

68. COLUMN 18. *Velocity due to the fall acting upon the wheel.* The numbers in this column have been calculated by the formula

$$V = \sqrt{2gh}.$$

V = the velocity in feet per second

g = the velocity acquired by a body at the end of the first second of its fall in a vacuum.

h = the fall acting upon the wheel; this is given in column 13.

The value of g has been calculated by the formula given in the second edition of the *Traité D'Hydraulique*, by *D'Aubuisson*, page 5, viz.:—

$$g = 9.^{m}8051\,(1 - 0.000284\cos 2l)\left(1 - \frac{2e}{r}\right);$$

l being the latitude of the place; e, its elevation above the level of the sea; and r, the radius of the terrestrial spheroid, at the level of the sea, and at the place;

$$\{\, r = 6366407^{m}\,(1 + 0.00164\cos 2l) \,\}.$$

The latitude of Lowell, [redacted] given in the American Almanac, is 42°, 38′, 46″, and the height above the sea is known to be about 25 metres. With these data, the above formula gives, in feet,

$$g = 32.1618$$

69. Column 19. *Velocity of the interior circumference of the wheel.* The diameter of the circle inscribing the inner edges of the buckets, is 6.75 feet; see art. 35. Consequently the interior circumference of the wheel is 21.20575 feet. The product of this number into the number of revolutions per second, given in column 8, gives the numbers in column 19.

70. Column 20. *Ratio of the velocity of the interior circumference of the wheel, to the velocity due to the fall acting* [redacted] *the wheel.* This column is obtained by dividing the numbers in column 19 by the corresponding numbers in column 18. This column indicates the relative velocities of the wheel, in the different experiments, eliminated from the effects of the variations in the fall acting upon the wheel.

71. Column 21. *Quantity of water which passed the wheel, reduced to* [redacted] *uniform fall of thirteen feet.* The numbers in this column are obtained from those in column 15, in the following manner.

Let H = the observed fall acting upon the wheel.
Q = the observed quantity.
Q' = the quantity that would have passed the wheel, if the fall had been thirteen feet, instead of H, all other circumstances being the same.

As the quantity of water discharged by the wheel, all other things being equal, will vary [redacted] the square root of the fall acting upon the wheel, we have

$$\sqrt{H} : Q : \sqrt{13} : Q',$$

therefore

$$Q' = Q\sqrt{\frac{13}{H}}.$$

The quantities given in column 21, have been calculated by this formula.

72. Column 22. *Ratio of the reduced quantity in column 21, to the reduced quantity in experiment* 30. The numbers given in this column indicate the relative quantities discharged by the wheel in the different experiments, eliminated from the effects due to the variations in the fall acting upon the wheel; the reduced quantity in experiment 30 is taken as unity, that experiment giving the maximum coefficient of effect. It will be seen by a comparison of columns 20 and 22, that the quantity discharged by the wheel, when the gate is fully raised, diminishes regularly with the velocity. The quantity discharged is a minimum in experiment 42,

in which the wheel had the least velocity. In experiments 43 and 44, however, in which the wheel was prevented from revolving, by screwing up the brake, the quantity discharged was considerably above the minimum. Whether this is due to an accidental position of the buckets relative to the guides, presenting apertures more favorable to the discharge than the average of all positions, or whether it is due to some more general cause, the author is not aware.

73. COLUMN 23. *Direction of the water leaving the wheel, as indicated by the vane* The angles given in this column show the position of the vane, relative to a line passing through the axis of the vane, and parallel to a tangent drawn through the point in the circumference of the wheel, nearest to the axis of the vane, zero being in the direction of the motion of the wheel. The apparatus with which these angles were taken, is described at art. 43. In the experiments made when the gate was fully raised, or nearly so, the vane operated satisfactorily; but as the height of the gate was diminished, the indications of the vane became more uncertain. The vane was of nearly the same height as the orifices in the exterior circumference of the wheel; this was very suitable for the experiments with the gate fully raised, but in the experiments with the gate partially raised, a portion of the height of the vane, was exposed to irregular currents, which probably interfered seriously with its operation. The observations made with the vane in the experiments in which the gate was partially raised, are much less to be relied on than those made when the gate was fully raised, the value of the indications being, in some degree, proportioned to the height of the gate.

74. COLUMN 24. *Mean elevation of the pointer on the bell crank* The numbers in this column indicate the mean positions of the bell crank, during the experiments, in reference to a gauge placed 6.5 feet from the fulcrum of the bell crank. It will be seen by the table that the mean positions differ but little from the horizontal; the pointer was however generally a little below, which indicates that the weight was generally lifted a little too high

The play of the brake was confined between two fixed stops, placed so that when the pointer stood at 0.20 feet below the horizontal, the brake struck,—and it struck the other stop when the pointer was at 0.21 feet above the horizontal. The brake was not allowed, however, to touch either of the stops in any of the experiments reported, in which it was undertaken to regulate the friction of the brake; the fact that it did touch was deemed a sufficient reason to reject the experiment. Little inconvenience, however, was experienced from this cause, as the hydraulic regulator afforded very perfect control over the brake.

TABLE

EXPERIMENTS UPON THE TURBINE AT THE

1	2	3		4	5						[illegible]	7	8	9	10
No. of the experiment	DATE, 1851.	Temperature of the atmosphere in degrees of Fahrenheit's thermometer		Height of the regulating gate in inches	TIME						Duration of the experiment, in seconds	Total number of revolutions of the wheel during the experiment	Number of revolutions of the wheel per second	Weight in the scale, in pounds avoirdupois	Useful effect, or the friction of the brake, in pounds avoirdupois raised one foot per second
		External air in the shade.	In the wheelpit		Beginning of the experiment			End of the experiment							
					H.	m.	sec.	H.	m.	sec.					
1	February 17, P.M.	31.00	41.00	11.50	2	19	52.00	2	28	15.50	503.50	450	0.89374	1443.84	87760.8
2	" " "	29.00	36.75	11.60	2	37	5.00	2	47	21.75	616.75	550	0.89177	1443.84	87567.2
3	" " "	30.25	36.25	11.45	2	56	26.00	3	6	41.50	615.50	550	0.89358	1443.34	87715.0
4	" " "	29.00	35.25	11.49	[illegible]	23	24.00	3	29	8.50	344.50	550	1.59651	507.03	55518.3
5	" " "	29.50	35.50	[illegible]	3	29	8.50	3	37	18.00	489.50	750	1.53218	411.48	42892.0
6	" " "	29.75	35.25	"	3	37	18.00	3	44	42.75	444.75	650	1.46149	514.77	51080.0
7	" " "	29.50	35.50	"	3	45	18.00	3	52	32.00	434.00	600	1.38249	634.86	60040.8
8	" " "	29.25	35.50	"	4	4	35.00	4	9	40.50	305.50	400	1.30933	750.42	66845.5
9	" " "	29.25	35.50	"	4	10	19.75	4	15	41.00	321.25	400	1.24514	854.87	72416.3
10	" " "	29.00	35.50	"	4	15	41.00	4	24	7.50	506.50	600	1.18460	957.85	77184.8
11	" " "	29.00	35.75	"	4	21	51.00	4	30	44.25	533.25	600	1.12518	1057.49	[illegible]19.8
12	" " "	28.50	35.00	"	5	1	10.50	5	10	31.00	560.50	1000	1.78412	0.	0.
13	February 18, A.M.		35.75	"	9	14	5.50	9	22	58.00	532.50	950	1.78404	0.	0.
14	" " "	33.75	36.25	"	9	42	32.00	9	51	7.25	515.25	550	1.06744	1155.27	83909.8
15	" " "	34.25	36.75	"	9	51	7.25	10	0	4.50	537.25	550	1.02374	1229.41	85625.9
16	" " "	34.00	36.50	"	10	12	27.00	10	19	57.25	450.25	450	0.99945	1269.42	86311.1
17	" " "	33.75	36.50	"	10	20	48.00	10	29	23.50	515.50	500	0.96993	1319.22	87051.8
18	" " "	34.00	36.50	"	10	41	55.00	10	48	58.25	423.25	400	0.94507	1359.28	87392.7
19	" " "	34.75	36.75	"	10	49	52.00	10	59	48.00	596.00	550	0.92282	1397.12	87711.1
20	" " "	36.00	36.00	"	11	16	14.50	11	25	23.25	548.75	500	0.91116	1416.70	87810.8
21	" " "	36.50	36.50	"	11	25	34.25	11	35	44.00	609.75	550	0.90201	1433.48	87961.3
22	" " "	36.50	36.75	"	11	45	12.00	11	59	8.00	836.00	750	0.89713	1443.06	88070.2
23	February 18, P.M.	41.50	39.25	"	[illegible]	23	56.50	2	33	18.00	561.50	1000	1.78094	0.	0.
24	" " "	39.75	38.75	"	[illegible]	41	30.50	2	50	51.00	560.50	500	0.89206	1454.21	88257.1
25	" " "	39.00	40.00	"	[illegible]	50	51.00	3	4	2.00	791.00	700	0.88496	1464.80	88183.9
26	" " "	38.75	38.00	"	3	22	7.50	3	26	53.25	285.75	250	0.87489	1474.87	87736.6
27	" " "	38.75	38.00	"	3	27	54.00	3	42	6.25	852.25	750	0.88002	1474.87	88271.1
28	" " "	38.50	36.25	"	[illegible]	58	40.25	4	11	4.75	744.50	650	0.87307	1485.63	88242.7
29	" " "	38.50	36.25	"	4	28	54.50	4	40	27.00	692.50	600	0.86643	1498.68	88339.3
30	" " "	37.25	36.50	"	4	55	58.00	5	7	43.00	705.00	600	0.85106	1524.67	88278.9
31	February 20, A.M.			11.48	9	16	16.25	9	25	35.00	558.75	1000	1.78971	0.	0.
32	" " "	33.50	35.00	"	9	47	40.50	9	59	39.25	678.75	600	0.88624	1552.41	88890.8
33	" " "	33.75	35.00	"	10	[illegible]	35.00	10	18	49.75	611.75	500	0.81831	1597.08	88872.5
34	" " "	36.75	35.50	"	10	37	30.50	10	48	8.25	637.75	500	0.78401	1618.87	87947.8
35	" " "	38.00	35.75	"	11	[illegible]	8.50	11	18	22.25	673.75	500	0.74211	1721.49	87076.5
36	" " "	41.00	35.75	"	11	28	38.25	11	39	26.00	647.75	450	0.69471	1816.71	85801.7
37	" " "	41.50	36.00	"	11	48	56.00	11	59	15.50	619.50	400	0.64568	1911.45	88063.5
38	February 20, P.M.			"	[illegible]	30	39.50	2	39	59.50	560.00	1000	1.78571	0.	0.
39	" " "	42.25	35.25	"	2	55	11.00	3	6	46.50	695.50	450	0.64702	1911.15	84130.1
40	" " "	41.75	35.75	"	3	21	56.50	3	30	16.50	500.00	300	0.60000	2011.52	82100.8
41	" " "	41.75	35.75	"	[illegible]	45	27.50	3	56	25.00	657.50	350	0.53232	2167.98	78492.2
42	" " "	41.50	36.00	"	4	9	29.00	4	18	39.50	550.50	250	0.45413	2367.88	73158.0
43	" " "			"	4	58	0.	4	59	30.00	90.00	0	0.	4212.38	0.
44	" " "			"	[illegible]	[illegible]	0.	5	4	30.00	150.00	0	0.	3946.98	0.
45	February 21, A.M.	36.25	35.50	"	[illegible]	22	57.50	9	32	18.25	560.75	1000	1.78333	0.	0.
46	" " "	36.25	35.75	"	9	38	3.00	9	49	4.00	661.00	550	0.83207	1567.21	88603.9
47	" " "	36.25	35.75	"	10	0	48.75	10	11	1.00	612.25	500	0.81666	1596.50	88867.8
48	" " "	35.75	36.25	"	10	25	38.50	10	37	8.25	684.75	550	0.80321	1614.79	88210.1
49	" " "	36.00	36.00	"	10	50	35.00	11	2	11.75	696.75	550	0.78938	1641.34	88146.2
50	" " "	35.75	36.25	"	11	14	54.00	11	25	44.25	650.25	500	0.76893	1671.02	87875.8

II.

TREMONT MILLS, IN LOWELL, MASSACHUSETTS

No. of the experiment	11	12	13	14	15	16	17	18	19	20	21	22	23		24
	Height of the water above the wheel, near the flume, in feet	Height of the water after passing the wheel, in feet	Total fall acting upon the wheel, in feet	Depth of water on the weir, in feet	Quantity of water which passed the wheel, in cubic feet per second	Total power of the water in pounds raised one foot per second	Ratio of the useful effect to the power expended	Velocity due to the fall acting on the wheel, in feet per second	Velocity of the exterior circumference of the wheel, in feet per second	Ratio of the velocity of the exterior circumference of the wheel to the velocity due to the fall acting on the wheel	Quantity of water which passed the wheel, reduced to a uniform fall of 13 feet, in cubic feet per second	Ratio of the reduced quantity in column 21 to the reduced quantity in experiment 30	Direction of the water leaving the wheel, as indicated by the vane		Mean elevation of the water on the [illegible] tank.
													deg	m	Feet.
1	15.082	2.218	12.861	1.8811	139.4206	111870.0	0.78440	28.7650	18.9525	0.65886	140.1557	1.01011	47	0	+0.002
2	15.079	2.219	12.860	1.8811	139.4206	111835.2	0.78300	28.7611	18.9107	0.65751	140.1775	1.01060	47	15	0.
3	15.087	2.218	12.869	1.8816	139.4676	111951.2	0.78378	28.7712	18.9491	0.65861	140.1756	1.01058	47	23	—0.001
4	15.036	2.482	12.554	2.0883	160.6470	122668.8	0.27187	28.4169	33.8558	1.19138	159.4058	1.14922			+0.018
5	15.042	2.481	12.561	2.0180	154.8891	121444.2	0.35318	28.4819	32.4909	1.14078	156.7522	1.13000			+0.008
6	15.051	2.393	12.658	1.9989	152.2682	120174.8	0.43005	28.5287	30.9021	1.08[illegible]35	154.3121	1.11271			—0.008
7	15.061	2.365	12.696	1.9784	149.4058	118308.5	0.50726	28.5771	29.3167	1.02588	151.2442	1.09038	15	40	—0.001
8	15.069	2.349	12.720	1.9586	147.2942	116804.8	0.57199	28.6041	27.7653	0.97067	148.9000	1.07353	18	8	—0.010
9	15.089	2.312	12.777	1.9420	146.0204	116378.2	0.62228	28.6681	26.4041	0.92102	147.289	1.06187	20	8	—0.018
10	15.102	2.302	12.800	1.9315	144.8734	115667.0	0.66704	28.6989	25.1208	0.87540	146.0003	1.05258	22	56	—0.018
11	15.100	2.281	12.819	1.9226	143.9088	115067.6	0.70850	28.7152	23.8602	0.83098	144.9212	1.04480	25	47	—0.001
12	15.028						0.		37.8890						
13	15.071	2.561	12.510	2.0989	163.4818	127527.8	0.	28.3670	37.8310	1.33366	166.6013	1.20110			
14	15.120	2.261	12.859	1.9098	142.5180	114281.2	0.73[illegible]75	28.7566	22.6359	0.78716	143.3110	1.03321	20	40	+0.002
15	15.117	2.229	12.888	1.9054	142.0438	114187.1	0.74[illegible]87	28.7921	21.7000	0.75398	142.6502	1.02849	33	30	+0.001
16	15.116	2.226	12.890	1.9048	141.9762	114150.8	0.75614	28.7946	21.1940	0.73604	142.5808	1.02792	35	37	—0.001
17	15.128	2.232	12.896	1.8988	141.2702	113640.9	0.76608	28.8018	20.5681	0.71414	141.8448	1.02262	38	20	—0.001
18	15.111	2.231	12.880	1.8908	140.4657	112848.8	0.77[illegible]42	28.7885	20.0400	0.69626	141.1180	1.01738	41	26	—0.008
19	15.111	2.231	12.883	1.8878	140.0815	112568.7	0.77920	28.7868	19.5691	0.67979	140.7192	1.01450	44	26	+0.001
20	15.111	2.228	12.883	1.8866	140.0000	112532.8	0.78010	28.7902	19.3219	0.67118	140.6240	1.01382	46	18	—0.002
21	15.128	2.229	12.899	1.8856	139.9010	112508.8	0.78147	28.8047	19.1278	0.66405	140.4507	1.01257	47	26	—0.005
22	15.123	2.225	12.898	1.8834	139.6078	112364.5	0.78884	28.8036	19.0248	0.66048	140.2190	1.01090	48	26	0.
23	14.[illegible]5	2.536	12.429	2.0831	161.6344	125355.0	0.	28.2750	37.7602	1.33567	165.3609	1.19220			
24	15.12	2.219	12.905	1.8773	139.0070	111808.5	0.78876	28.8114	18.9168	0.65657	139.5177	1.00584	49	28	—0.006
25	15.118	2.219	12.899	1.8775	139.0291	111859.4	0.78810	28.8047	18.7662	0.65150	139.5724	1.00628	50	37	+0.001
26	15.102	2.209	12.893	1.8750	138.7601	111591.0	0.78611	28.7980	18.5527	0.64424	139.3347	1.00452	[illegible]	40	—0.057
27	15.116	2.214	12.902	1.8758	138.8189	111740.9	0.78997	28.8080	18.6616	0.64779	139.3752	1.00481	51	18	—0.018
28	15.117	2.211	12.906	1.8740	138.8711	111702.9	0.78934	28.8125	18.5141	0.64257	139.3760	1.00482	52	25	—0.018
29	15.118	2.212	12.906	1.8727	138.5134	111501.9	0.79225	28.8125	18.3782	0.63708	139.0109	1.00228	53	52	—0.017
30	15.111	2.208	12.903	1.8697	138.1892	111218.1	0.79375	28.8092	18.0474	0.62645	138.7076	1.00000	58	10	—0.018
31	15.[illegible]8	2.515	12.583	2.0891	162.8283	126060.2	0.	28.8990	37.0521	1.33685	165.2853	1.19161			
32	15.119	2.201	12.918	1.8704	138.2608	111384.0	0.79204	28.8225	17.7330	0.61525	138.7211	1.00010	61	51	—0.011
33	15.131	2.200	12.931	1.8701	138.2385	111521.1	0.79248	28.8437	17.2475	0.59796	138.5858	0.99912	66	5	—0.008
34	5.129	2.188	12.941	1.8687	138.0809	111408.0	0.78908	28.8515	16.6254	0.57624	138.4013	0.99770	80	12	—0.001
35	15.123	2.184	12.939	1.8652	137.7076	111189.7	0.78810	28.8498	15.7871	0.5[illegible]	138.0818	0.99513	90	25	—0.014
36	15.[illegible]28	2.184	12.944	1.8539	136.4017	110200.9	0.77916	28.8540	14.7319	0.51056	136.7866	0.98615	115	48	—0.001
37	15.117	2.177	12.940	1.8412	135.1415	109077.1	0.76978	28.8504	13.6022	0.47459	135.4545	0.97655	131	18	—0.018
38	15.036	2.536	12.500	2.0884	161.6914	126071.1	0.	28.3557	37.8674	1.33544	164.8960	1.18881			
39	15.143	2.180	12.963	1.8481	135.8428	109483.4	0.76886	28.8761	13.7205	0.47515	135.5954	0.97718	130	51	—0.010
40	15.136	2.163	12.973	1.8380	134.7970	109077.0	0.75277	28.8872	12.7285	0.44045	134.9378	0.97282	139	15	—0.010
41	15.[illegible]36	2.159	12.977	1.8282	133.7588	108265.8	0.72499	28.8916	11.2882	0.39071	133.8728	0.96514	147	25	—0.005
42	15.133	2.185	12.948	1.8252	133.4880	107764.7	0.67887	28.8508	9.6802	0.33570	133.7007	0.96390			—0.015
43	15.[illegible]16	2.319	12.797	1.8160	135.6536	108280.4	0.	28.6906	0.	0.	136.7258	0.98571			
44	15.096	2.322	12.774	1.8457	135.6205	108059.4	0.	28.6648	0.	0.	136.8150	0.98635			
45	15.012	2.541	12.471	2.0864	162.0297	126034.8	0.	28.3228	37.8168	1.33521	165.4245	1.19261			
46	15.156	2.202	12.954	1.8787	138.6214	112009.8	0.79104	28.8660	17.6447	0.61126	138.8703	1.00117	60	52	—0.012
47	15.131	2.202	12.932	1.8726	138.5023	111720.6	0.79097	28.8415	17.3179	0.60046	138.8660	1.00114	68	16	—0.008
48	15.142	2.194	12.948	1.8723	138.4090	111882.0	0.78904	28.8598	17.0827	0.59020	138.7468	1.00028	66	27	—0.016
49	15.141	2.190	12.951	1.8714	138.3692	111777.2	0.78859	28.8627	16.7394	0.57997	138.6307	0.99945	81	48	—0.007
50	15.144	2.192	12.952	1.8694	138.1559	111618.5	0.78723	28.8698	16.3058	0.56492	138.4117	0.99787	89	44	—0.010

TABLE

EXPERIMENTS UPON THE TURBINE AT THE

1	2	3		4	5						6	7	8	9	10
No. of the experiment	DATE 1851	Temperature of the atmosphere in degrees of Fahrenheit's thermometer		Height of the regulating gate, in inches	TIME						Duration of the experiment, in seconds	Total number of revolutions of the wheel during the experiment	Number of revolutions of the wheel per second	Weight in the scale, in pounds avoirdupois	Useful effect, or the friction of the brake, in pounds raised one foot per second
		External air in the shade	In the wheel-pit		Beginning of the experiment			Ending of the experiment							
					H	Min	Sec	H	Min	Sec					
51	February 21, P.M.	34.75	36.50	8.55	2	17	11.50	2	23	58.00	401.50	600	1.48131	890.95	39152.1
52	" " "	34.50	36.25	"	2	24	31.00	2	32	28.00	477.00	600	1.2578[illegible]	775.61	66473.6
53	" " "	34.50	36.50	"	2	33	10.00	2	41	55.50	525.50	600	1.14177	964.30	71827.2
54	" " "	34.50	34.50	"	2	41	57.50	2	50	25.00	509.50	550	1.07949	1069.05	78512.0
55	" " "	34.50	37.00	"	2	50	25.00	2	58	33.00	488.00	500	1.02459	1150.77	80218.4
56	" " "	[illegible]	36.75	"	3	8	31.00	3	16	20.50	469.50	450	0.95847	1242.98	81572.6
57	" " "	34.50	37.00	"	3	17	13.50	3	25	17.00	483.50	450	0.93071	1263.09	81911.6
58	" " "	34.25	37.00	"	3	25	17.00	3	33	37.00	500.00	450	0.90000	1315.17	82382.7
59	" " "	34.25	37.25	"	3	33	37.00	3	42	17.00	520.00	450	0.86538	1366.11	82195.5
60	" " "	34.25	36.75	"	3	[illegible]	[illegible]	3	52	14.25	518.75	450	0.86527	1411.03	82057.9
61	" " "	34.25	34.50	"	4	[illegible]	[illegible]	4	11	45.50	[illegible]	450	[illegible]	[illegible]	81786.2
62	" " "	34.00	36.00	"	4	31	37.50	4	40	15.50	518.00	400	0.77220	1518.69	81360.8
63	" " "	34.00	36.00	"	4	51	21.50	4	59	36.00	494.50	350	0.70779	1656.06	79788.1
64	" " "			"	5	8	37.00	5	19	17.25	640.25	1100	1.71808	0.	0.
65	February 22, A.M.			5.05	8	57	1.00	9	7	47.00	646.00	1000	1.54799	0.	0.
66	" " "	35.50	35.75	"	9	15	39.00	9	21	12.00	333.00	450	1.35135	316.05	29054.7
67	" " "	35.50	36.50	"	9	21	51.25	9	28	0.50	369.25	450	1.21869	519.60	43087.9
68	" " "	35.50	36.75	"	9	28	45.00	9	36	26.00	461.00	500	1.08460	720.20	53142.1
69	" " "	35.50	37.25	"	9	37	26.00	9	43	4.50	398.50	400	1.00376	892.26	56834.2
70	" " "	35.75	37.25	"	9	43	57.00	9	50	16.00	379.00	350	0.92348	993.71	58727.1
71	" " "	35.75	37.25	"	9	51	14.00	9	58	9.00	415.00	350	0.84337	1033.80	59287.8
72	" " "	36.75	37.25	"	9	59	12.50	10	7	48.75	516.25	400	0.77482	1115.02	58770.2
73	" " "	39.00	35.75	"	10	21	10.00	10	30	50.25	580.25	450	0.77553	1115.02	58840.1
74	" " "	38.25	36.00	"	10	42	[illegible]	10	51	14.50	510.00	350	0.68627	1203.81	56273.1
75	" " "	38.25	36.25	"	10	58	28.00	11	6	35.00	487.00	300	0.61602	1277.98	54559.1
76	" " "	38.25	36.25	"	11	13	59.00	11	21	17.00	438.00	200	0.45662	1482.50	[illegible]
77	February 22, A.M.	37.50	35.75	9.96	11	33	20.00	11	40	9.50	409.50	350	0.85470	1482.50	86907.0
78	" " "	38.00	36.00	"	11	46	21.00	11	53	27.00	426.00	350	0.82160	1544.87	86151.1
79	" " P.M.	40.75	35.75	"	0	0	42.50	0	[illegible]	6.00	413.50	350	0.78018	1601.85	86101.4
80	February 22, P.M.	42.75	36.00	2.875	2	38	11.00	2	43	31.25	320.25	400	1.24902	0.	0.
81	" " "	43.75	36.50	"	2	42	8.00	2	47	52.00	349.00	400	1.14613	118.50	9217.0
82	" " "	44.00	37.00	"	2	48	41.00	2	54	45.50	364.50	350	0.96022	425.30	21258.6
83	" " "	44.25	37.25	"	3	55	16.00	3	[illegible]	14.50	388.50	300	0.77220	519.86	27310.9
84	" " "	43.75	37.25	"	3	2	14.50	3	9	41.00	446.50	300	0.67189	611.22	27985.1
85	" " "	43.50	37.00	"	3	11	21.50	3	18	48.00	446.50	250	0.55991	70[illegible].14	26893.8
86	" " "	43.50	36.75	"	3	20	34.50	3	27	47.00	432.50	200	0.46243	777.58	21162.9
87	" " "	43.25	36.50	"	3	33	10.50	3	44	17.00	666.50	200	0.30007	882.02	18006.4
88	" " "	42.25	36.25	"	4	2	0.					0	0.	1197.06	0.
89	" " "			"	4	10	0.					0	0.	105[illegible].25	0.
90	February 22, P.M.	42.00	37.75	1.00	4	27	3.50	4	33	47.00	403.50	250	0.61958	118.59	4998.8
91	" " "			"	4	35	1.00	4	42	16.50	435.50	300	0.68886	73.14	3127.7
92	" " "	41.00	37.25	"	4	45	12.00	5	2	51.00	1032.00	400	0.38760	296.39	7815.0

II. — CONTINUED

TREMONT MILLS, IN LOWELL, MASSACHUSETTS

No. of the experiment	11 Height of the water above the wheel, taken in the flume, in feet.	12 Height of the water after passing the wheel, taken in the wheel-pit, in feet.	13 Total fall acting upon the wheel, in feet.	14 Depth of water on the weir, in feet.	15 Quantity of water which passed the wheel, in cubic feet per second.	16 Total power of the water, in pounds raised one foot per second.	17 Ratio of the useful effect to the power expended.	18 Velocity due to the fall acting upon the wheel, in feet per second.	19 Velocity of the exterior circumference of the wheel, in feet per second.	20 Ratio of the velocity of the exterior circumference of the wheel to the velocity due to the fall acting upon the wheel.	21 Quantity of water which passed the wheel, reduced to a uniform fall of 13 feet, in cubic feet per second.	22 Ratio of the reduced quantity in column 21, to the reduced quantity in experiment 30.	23 Direction of the water as it leaves the wheel, indicated by the vane.		24 Mean elevation of the water in the pit, above the bench mark.
													deg	m	Feet
51	15.095	2.337	12.758	1.9178	143.3319	114000.7	0.34589	28.6468	31.4518	1.09802	144.6849	1.01809	12	0	+0.002
52	15.128	2.255	12.873	1.8792	139.2094	111778.7	0.59879	28.7751	26.0739	0.[illegible]2696	139.8945	1.00856	17	32	+0.001
53	15.184	2.225	12.959	1.8656	137.7518	110917.6	0.67162	28.8159	24.2121	0.84023	138.2365	0.99660	22	19	+0.007
54	15.134	2.194	12.940	1.8000	137.7902	111219.8	0.70592	28.8504	22.8014	0.79346	138.1158	0.99578	25	0	+0.013
55	15.139	2.189	12.950	1.8586	137.002	110664.7	0.72485	28.8610	21.7272	0.75281	137.2668	0.98961	28	56	—0.011
56	15.138	2.187	12.951	1.8450	135.5134	109494.5	0.74199	28.8627	20.1557	0.70873	135.7990	0.97903	33	58	—0.010
57	15.148	[illegible].178	12.965	1.8408	135.0974	109252.2	0.74975	28.8788	19.7365	0.68344	135.2797	0.97529	37	47	—0.011
58	15.141	[illegible].168	12.973	1.8336	134.3904	108724.1	0.75772	28.8905	19.0852	0.66060	134.4546	0.96934	42	48	—0.011
59	15.151	2.152	12.999	1.8240	133.301	108082.5	0.76049	28.9161	18.3511	0.63463	133.3066	0.96106	47	30	—0.005
60	15.153	2.139	13.014	1.8183	132.73[illegible]4	107746.9	0.76158	28.9828	17.7[illegible]26	0.[illegible]12[illegible]	132.6680	0.95642	51	37	—0.00[illegible]
61	15.155	2.129	13.026	1.8117	131.9360	107246.3	0.76260	28.9401	17.0550	0.58922	131.8642	0.95066	50	39	—0.012
62	15.162	2.122	13.040	1.8022	130.9913	106544.4	0.76868	28.9617	16.3751	0.56541	130.7901	0.94292	76	36	—0.021
63	15.162	2.134	13.028	1.8019	130.8932	106366.0	0.75012	28.9181	15.0091	0.51818	130.7525	0.94265	94	4	—0.022
64	15.079	2.359	12.720	1.9742	149.5470	118652.1	0.	28.6041	36.4382	1.27370	151.1840	1.08995			
65	15.139	1.969	13.170	1.7160	121.9685	100194.6	0.	29.1057	32.8262	1.12783	121.1788	0.87368			
66	15.148	2.071	13.077	1.6829	118.6511	96780.5	0.30040	29.0028	28.6564	0.98806	118.2010	0.85216	0	30	+0.001
67	15.159	2.025	13.134	1.6590	116.0987	95112.0	0.45302	29.0659	25.8482	0.88912	115.5050	0.83272	8	30	+0.002
68	15.16	1.988	13.176	1.6409	11[illegible].2599	93004.9	0.56592	29.1128	22.9997	0.79008	113.4042	0.81823	12	32	—0.020
69	15.171	1.956	13.215	1.6309	113.21[illegible]8	933[illegible]6.0	0.60885	29.1554	21.2850	0.73007	112.3198	0.80976	16	5	—0.027
70	15.174	1.920	13.253	1.6133	111.5197	92188.4	0.63708	29.1978	19.5331	0.67072	110.4502	0.79328	21	17	—0.006
71	15.179	1.897	13.282	1.5959	109.7130	90893.3	0.65228	29.2292	17.8844	0.61187	108.5420	0.78252	29	56	—0.014
72	15.183	1.872	13.311	1.5709	108.0452	89707.1	0.65620	29.2611	16.4800	0.56152	106.7756	0.76979	30	2	+0.001
73	15.179	1.869	13.310	1.5789	107.2498	89620.7	0.650[illegible]3	29.2600	[illegible]	0.53205	106.6848	0.76918	40	13	0.029
74	15.[illegible]50	1.833	13.326	1.5541	105.5911	87720.9	0.64127	29.2770	14.5580	0.49707	104.2858	0.75147	69	27	—0.028
75	15.174	1.812	13.362	1.5371	103.8510	86555.6	0.61879	29.3171	13.0631	0.44558	102.4852	0.73830	65	0	—0.024
76	15.183	1.771	13.412	1.5031	100.5410	84110.0	0.54757	29.3719	9.6880	0.32960	98.9847	0.71362	14[illegible]	56	—0.029
77	15.079	2.196	12.883	1.8620	137.8618	110880.[illegible]	0.78100	28.7808	18.1246	0.62961	137.0842	0.99478	55	52	—0.007
78	15.079	2.183	12.896	1.8688	136.9694	110176.6	0.78375	28.8013	17.4226	0.60492	137.5206	0.99144	61	0	—0.021
79	15.087	2.175	12.912	1.8541	136.5469	109978.0	0.78350	28.8192	16.7351	0.58069	137.0114	0.98777	71	28	—0.018
80	14.774	1.427	13.347	1.2914	80.4594	66979.0	0.	29.3006	26.4865	0.90396	79.4007	0.57243	0	30	
81	14.769	1.400	13.369	1.2787	78.8183	65746.8	0.14065	29.3248	24.3046	0.82881	77.7176	0.56051	1	30	+0.008
82	14.772	1.377	13.395	1.2492	76.6213	64018.1	0.34204	29.3533	20.3622	0.69369	75.4881	0.54419	4	32	+0.010
83	14.783	1.348	13.435	1.2200	74.0590	62062.0	0.41006	29.3971	16.3751	0.55708	72.8501	0.52521	11	30	+0.002
84	14.793	1.315	13.478	1.1960	71.8750	60121.6	0.46314	29.4441	14.2480	0.48390	70.5889	0.50890	20	0	—0.001
85	14.806	1.293	13.513	1.1748	70.0069	59006.4	0.45476	29.4823	11.8733	0.40273	68.6646	0.49508	41	34	—0.022
86	14.820	1.264	13.556	1.1497	67.8158	57842.0	0.42061	29.5292	9.8061	0.33208	66.4105	0.47878	81	40	—0.027
87	14.803	1.214	13.589	1.1119	64.5058	54554.9	0.33006	29.5324	6.3683	0.21547	63.1616	0.45536			—0.020
88	14.762	1.246	13.516	1.0623	60.3598	50886.6	0.	29.4856	0.	0.	59.1950	0.42677			
89	14.771	1.240	13.531	1.0630	60.4190	50998.4	0.	29.5020	0.	0.	59.2216	0.42695			
90	14.806	0.821	13.985	0.7798	38.2210	33340.8	0.14093	29.9928	13.1986	0.43806	36.8505	0.26567			+0.004
91	14.815	0.814	14.001	0.7840	38.5699	33688.5	0.10176	30.0093	14.6079	0.48677	37.1655	0.26794			+0.030
92	14.832	0.812	14.020	0.7653	37.1738	32508.0	0.21042	30.0303	8.2193	0.27370	35.7956	0.25806			—0.006

DESCRIPTION OF THE DIAGRAM REPRESENTING THE EXPERIMENTS.

75. For the purpose of presenting a general view of the experiments, the coefficients of effect, at different velocities, are plotted at figure 1, plate VI, on a system of coördinates. The ratios of the velocities of the interior circumference of the wheel, to the velocities due the fall acting upon the wheel, given in column 20, table II, are taken to represent the velocities; these ratios are here called the velocities, and are taken on the axis of abscissas *AX*; the corresponding coefficients of effect given in column 17, table II, are taken upon the axis of ordinates *AY*.

76. The line *CD* represents the experiments made with the regulating gate fully raised; — to avoid confusion a portion of the experiments are omitted; — the experiments represented are those numbered from 4 to 42, inclusive, which were made in regular sequence, with gradually increasing weights. It will be observed in the table of experiments, that several trials were made with the brake entirely removed; these were made, generally, after the wheel had been left for some time, for the purpose of seeing if it was in as good running order as usual; if any material change had taken place, it would have been indicated by a change in the velocity of the wheel.

The experiments thus made, omitting experiment 12, in which the height in the wheelpit was not observed, are collected together in the following table.

Number of the experiment.	Ratio of the velocity of the interior circumference of the wheel, to the velocity due to the fall acting upon the wheel.
13	1.33866
23	1.33517
31	1.31633
38	1.33511
45	1.34121
Mean . . .	1.33527

The greatest variation in these velocities is in experiment 13, which is $\frac{1}{70}$ part below the mean; the running condition of the wheel must, consequently, have been nearly uniform.

In all the experiments with the brake removed, the coefficient of effect, of course, is nothing, and they would be represented on the diagram by points on the axis of abscissas; for the sake of distinctness, only one of those tried when the gate was at its full height, is represented on the diagram.

There is a small irregularity in the line CD, at numbers 26 and 27; both these experiments were made with the same weight in the scale, and under similar circumstances, except that in 26, water was used to lubricate the friction pulley, and in 27 oil was used.

It has been stated, that, with heavy loads, the brake operates much more steadily with oil as a lubricator, than with water, and the change in the lubricator at experiment 27, was made in consequence of the difficulty experienced by the operator, in regulating the tension of the brake screws. In experiment 26, nearly his whole strength, applied to the extremity of a wrench about three feet long, was required to move the nuts, whereas, in experiment 27, the same operation was performed with great ease. Experiment 26 was of much shorter duration than experiment 27, and a portion of the discrepancy may be due to a proportionally less perfect observation of the data in 26.

The line CD shows that, with a velocity of the interior circumference of the wheel not less than 44 or more than 75 per cent of that due to the fall, the useful effect is 75 per cent. or more, of the total power expended. Beyond these points, the change in the coefficient of effect is nearly equal for equal and opposite variations of speed; thus, the diagram indicates that the coefficient of effect is 70 per cent. of the power expended, at the velocities 0.360 and 0.834.

$$0.436 - 0.360 = 0.076$$
$$0.834 - 0.750 = 0.084.$$

Taking the mean of the extreme velocities, that is, of 0, when the wheel was still, and 1.335, when the brake was removed, we have

$$\frac{1.335 + 0}{2} = 0.6675.$$

which is not far from the velocity giving the maximum coefficient of effect; that is to say, *when the gate is fully raised, the coefficient of effect is a maximum when the wheel is moving with about half its maximum velocity.*

77. Experiments 43 and 44 were both made with the gate fully raised, but the wheel at rest, the brake being screwed up sufficiently tight to prevent the wheel from revolving;—they were made for the purpose of ascertaining the total effort that could be exercised by the wheel.

By reference to column 9, of the table of experiments, it will be seen that, in experiment 43, the weight sustained was 4213.38 pounds, and in 44, the weight was 3946.38 pounds These experiments were made under circumstances nearly identical, except that in 43, the weight preponderated, and in 44, the power of the wheel preponderated. In 43, the weight was the least that would cause the scale to lower when the bell crank was placed horizontally, and then left free; on the other hand, in experiment 44, the weight was the greatest that would allow the scale to be raised under the same circumstances; that is to say, in 43, the weight represents the force exercised by the water against the wheel, *plus* the friction of the entire apparatus, and in 44, the weight represents the same thing, *minus* the friction; the difference of the weights, or 4213.88 — 3946.38 = 267 pounds, represents double the friction, and the true force exercised by the water against the wheel, is represented by the weight

$$\frac{4213.88 + 3946.38}{2} = 4079.88 \text{ pounds.}$$

This weight acted at a distance from the centre of the wheel, equal to the effective length of the brake, or 10.827778 feet (art. 50).

The radius of the turbine, at the outer extremities of the buckets, is 4.146 feet (art. 35), consequently, the equivalent force acting tangentially at the outer extremities of the buckets, was

$$\frac{4079.88 \times 10.827778}{4.146} = 10655.1 \text{ pounds.}$$

78. The line *EF* represents the experiments numbered 77, 78, and 79, made with the gate raised 9.96 inches, or about 87 per cent. of the full height. By a reference to the table of experiments, it will be seen that, although the regulating gate was lowered 13 per cent., the quantity of water discharged by the wheel was diminished less than one per cent.

79. The line *GH* represents the experiments numbered from 51 to 64, inclusive, made with the gate raised 8.55 inches, or about three fourths of the full height.

80. The line *IK* represents the experiments numbered from 65 to 76, inclusive, made with the gate raised 5.65 inches, or nearly a half of the full height.

81. The line *LM* represents the experiments numbered from 80 to 87, inclusive, made with the gate raised 2.875 inches, or one fourth of its full height. Experiments 88 and 89 were made with the same height of gate, but with the wheel held fast by the brake; the force exerted by the wheel at the distance 10.827778 feet, independent of friction, was

$$\frac{1195.06 + 1054.25}{2} = 1124.65 \text{ pounds.}$$

82. The line *NO* represents the three experiments numbered 90, 91, and 92, made with the regulating gate raised one inch.

An examination of the diagram will show that the velocity corresponding to the maximum coefficient of effect, diminishes with the height of the gate. For heights not less than one fourth of the whole height, this diminution is sufficiently regular; for heights less than one fourth, the experiments are not sufficient to indicate the velocity giving the best effect, but the diminution is evidently more rapid than for greater heights of gate.

PATH DESCRIBED BY A PARTICLE OF WATER IN PASSING THROUGH THE WHEEL

83. As in many other problems in hydraulics, resort is here had to a particular hypothesis, which, at best, is only an approximation to the truth, nevertheless, it may be the means of throwing some light upon the mode in which the water acts upon the wheel.

The particular hypothesis here assumed is this; *every particle of water contained in the wheel, situated at the same distance from the axis, moves in the same direction relative to the radius, and with the same velocity.* According to this hypothesis, the successive sections in which the same particles of water are found, are in cylindrical surfaces, concentric with the wheel.

Applying this hypothesis to experiment 30, on the Tremont Turbine, let us suppose

Q'' = the mean quantity of water discharged through each aperture of the wheel, in cubic feet per second.

ω = the angular velocity of the wheel.

R = the radius of the circle inscribing the inner edges of the buckets, or OA, figure 3, plate VI.

R' = the radius OB.

t = the time occupied by a particle of water in passing from the section AD to the section BC, or, which is the same thing, through the radial distance $R' - R$.

A = the area of $ABCD$, in square feet.

H = the mean height, in feet, between the crowns of the wheel, between the sections AD and BC.

We have

AH = the volume of water contained between the sections AD and BC.

t is the time occupied by a particle of water in passing from the section AD to the section BC, and it will evidently be the time required for the discharge of the volume AH. We find t by the proportion

$$Q'' : 1 :: AH : t = \frac{AH}{Q''}.$$

If the wheel was at rest, a particle of water at A would arrive at B in the time t, but the wheel is moving with the angular velocity ω, therefore the point B, in the time t, will have advanced to E, and

$$BE = R'\omega t = \frac{R'\omega AH}{Q''},$$

consequently, a particle of water at A, instead of being at B, at the end of the time t, will have arrived, by some path, at the point E. In this manner, by taking successive values of R', sufficiently near to each other, the entire path of a particle of water, from its entrance into the wheel, up to the moment of its discharge, may be traced; and as, by the hypothesis, all the particles at the same distance from the axis move with the same velocity, and in the same relative direction, the path of the entire stream, from its entrance into the wheel to its discharge, will be determined.

In experiment 30, we have the total quantity discharged by the wheel equal to 138.1802 cubic feet per second, as the wheel has forty-four apertures,

$$Q'' = \frac{138.1802}{44} = 3.14066 \text{ cubic feet per second.}$$

The velocity of the interior circumference of the wheel was 18.0474 feet per second, and the interior radius of the wheel being 3.375 feet, we have

$$\omega = \frac{18.0474}{3.375} = 5.3474 \text{ feet per second,}$$

consequently,

$$BE = \frac{5.3474\, R' AH}{3.14066} = 1.7026\, R' AH.$$

34. The successive steps in the calculation for the entire path, are given in table III.

The arcs of circles FG, HI, etc are drawn on a plan of the buckets, figure 2, plate VI., with the radii contained in the first column.

COLUMN 2 contains the entire areas of these circles.

COLUMN 3 contains the areas of the rings comprised between these circles, which are obtained by taking the differences of the successive areas in column 2.

COLUMN 4 contains the areas reduced to square feet, of that part of each ring corresponding to a single aperture in the wheel, including also the area occupied by the thickness of the corresponding part of one bucket

COLUMN 5. Corrections for the thickness of the buckets;—these are deduced from measurements taken on a full sized plan of the buckets.

COLUMN 6. True areas of the partial rings, being the differences of the corresponding areas in columns 4 and 5.

COLUMN 7. Mean heights of the partial rings;—these are also taken from a full sized drawing of the wheel.

COLUMN 8. Volumes of the partial rings, or the products of the corresponding numbers in columns 6 and 7.

COLUMN 9. Volumes between the radius R, and the successive values of the radius R'. These are obtained by adding together the volumes of the partial rings, up to the corresponding radius;—they are the successive values of AH.

COLUMN 10. The ordinates;—these are successive values of

$$1.7026\, R' A H,$$

the successive values of R' being taken in feet, instead of inches, as they are given in column 1.

TABLE III.

1 Values of R, and successive values of R' in inches.	2 Areas in square inches of circles of the radii in the last column.	3 Areas in square inches of the complete rings.	4 $\frac{1}{44}$ of the areas of the rings in last column, in square feet.	5 Corrections for the thickness of the buckets, in square feet.	6 True areas of the partial rings, in square feet.	7 Mean heights of the partial rings, in feet.	8 Volumes of the partial rings, in cubic feet.	9 Volumes between R and the successive values of R' in cubic feet.	10 Ordinates in feet, to be measured on axes of the corresponding radii in column 1.
40.5	5152.997								
41.5	5410.608	257.611	0.04066	0.00091	0.03975	0.9261	0.03682	0.03682	0.2168
42.5	5674.502	263.894	0.04165	0.00099	0.04066	0.9080	0.03692	0.07374	0.4447
43.5	5944.679	270.177	0.04264	0.00106	0.04158	0.8940	0.03717	0.11091	0.6845
44.5	6221.139	276.460	0.04363	0.00115	0.04248	0.8840	0.03755	0.14846	0.9373
45.5	6503.882	282.743	0.04462	0.00128	0.04334	0.8775	0.03803	0.18649	1.2039
46.5	6792.909	289.027	0.04562	0.00146	0.04416	0.8755	0.03866	0.22515	1.4851
47.5	7088.218	295.309	0.04661	0.00174	0.04487	0.8800	0.03949	0.26464	1.7835
48.5	7389.811	301.593	0.04760	0.00212	0.04548	0.8920	0.04057	0.30521	2.1003
49.0	7542.964	153.153	0.02417	0.00138	0.02279	0.9055	0.02064	0.32585	2.2654
49.25	7620.129	77.165	0.01218	0.00078	0.01140	0.9145	0.01042	0.33627	2.3498
49.50	7697.687	77.558	0.01224	0.00087	0.01137	0.9210	0.01047	0.34674	2.4352
49.75	7775.638	77.951	0.01230	0.00081	0.01149	0.9277	0.01066	0.35740	2.5228

85. The arcs FG, HI, etc., figure 2, plate VI, are taken equal to the ordinates 0.2168, 0.4447 etc, in column 10 of the table; the points Q, G, I, etc. K, are joined by a line, which is the limit of the stream on one side. The limit on the other side is found by making the arcs $GL = FN$, $IM = HO$, etc.; the points R, L, M, etc. P, being joined by a line, give the limits of the stream on this side.

86. By an inspection of the figure, it is plain that, in experiment 30, the path of the water through the wheel must have been a continuation of the direction given to it by the fixed guides VW, and that there was no sudden change of direction or velocity, up to a point near where the water was discharged from the wheel. The abrupt change at this point, indicated by the figure, could not, in reality, have taken place, as we know by the direction assumed by the vane, which is represented at ST in its mean position during the experiment.

87. The foregoing hypothesis will evidently lead to results more nearly correct, the nearer the buckets are to each other, until, in the case in which the spaces between them are infinitely small, it will give the path accurately. In applications like the above, where the spaces are very considerable, it is assumed by the hypothesis that the water passes through in curved laminæ, superimposed on each other, the first of which, in contact with the concavity of the bucket, is constrained by it and the rotation of the wheel, to move in a particular path; this, in its turn, constrains the next lamina to move in a similar path; and so on.

By an inspection of figure 2, plate VI, it is reasonable to suppose, that a lamina, far removed from the concavity of the bucket, will take a path differing from that of a lamina near it; the abruptness in the curve near its extremity, will be diminished, somewhat in proportion to the distance of the lamina from the concavity of the bucket, the water passing out from the wheel more nearly in the direction in which it was moving, during its approach to the circumference of the wheel. These views go far to explain the discrepancy between the path determined by the hypothesis, and the direction assumed by the vane.

88. Whatever objection may be made to the method by which the path, given in figure 2, plate VI., is obtained, it cannot be denied that its general course must have been nearly as represented; this being admitted, it is difficult to see how centrifugal force can operate in the important manner that is commonly assigned to it. The path is concave to the axis only in a very slight degree, and through a part only of its course; nevertheless, it is only in con-

sequence of a concavity in the path, that centrifugal force can have any existence With the gate only partially raised, this force may act powerfully in increasing the discharge, and a similar effect may be produced, at high velocities, with the gate fully raised; but in experiment 30, giving the maximum coefficient of effect, it can have had only [illegible] slight action.

RULES FOR PROPORTIONING TURBINES.

89. In making the designs for the Tremont, and other turbines, the author has been guided by the following rules, which he has been led to by a comparison of several turbines designed by Mr. Boyden, which have been carefully tested and found to operate well.

Rule 1st. The sum of the shortest distances between the buckets, should be equal to the diameter of the wheel

Rule 2d. The height of the orifices at the circumference of the wheel, should be equal to one tenth of the diameter of the wheel.

Rule 3d. The width of the crowns should be four times the shortest distance between the buckets.

Rule 4th. The sum of the shortest distances between the curved guides, taken near the wheel, should be equal to the interior diameter of the wheel.

The turbines, from a comparison of which the above rules were derived, varied in diameter from twenty-eight inches to nearly one hundred inches, and operated on falls from thirty feet to thirteen feet. The author believes that they may be safely followed for all falls between five feet and forty feet, and for all diameters not less than two feet, and, with judicious arrangements in other respects, and careful workmanship, a useful effect of seventy-five per cent. of the power expended, may be relied upon. For falls greater than forty feet, the second rule should be modified, by making the height of the orifices smaller in proportion to the diameter of the wheel.

90. Taking the foregoing rules as a basis, we may, by aid of the experiments on the Tremont Turbine, establish the following formulas.

Let $D=$ the diameter of the wheel at the outer extremities of the buckets.
$d=$ the diameter of the wheel, at the interior extremities of the buckets.
$H=$ the height of the orifices of discharge, at the outer extremities of the buckets.
$W=$ the width of the crowns occupied by the buckets.

N = the number of buckets.
n = the number of guides.
P = the horse-power of the turbine; a horse-power being 550 pounds avoir. raised one foot per second.
h = the fall acting upon the wheel.
Q = the quantity of water expended by the turbine, in cubic feet per second.
V = the velocity due the fall acting upon the wheel.
V' = the velocity of the water passing the narrowest sections of the wheel.
v = the velocity of the interior circumference of the wheel: all the velocities being in feet per second.
C = the coefficient of V, or the ratio of the real velocity of the water passing the narrowest sections of the wheel, to the theoretical velocity due the fall acting upon the wheel.

The unit of length is the English foot.

It is assumed that the useful effect is seventy-five per cent. of the total power of the water expended.

According to rule 1, we have the sum of the widths of the orifices of discharge, equal to D. Then the sum of the areas of all the orifices of discharge, is equal to DH.

By the fundamental law of hydraulics we have

$$V = \sqrt{2gh}.$$

therefore

$$V' = C\sqrt{2gh}.$$

We can find the value of C in the last equation by experiment 30, on the Tremont Turbine. In that wheel we have for the sum of the widths of the orifices of discharge, $44 \times 0.18757 = 8.25308$ feet, and the height of the orifices of discharge $= 0.9314$ feet. Then we have, for the sum of the areas of all the orifices of discharge,

$$HD = 8.25308 \times 0.9314 = 7.68692 \text{ square feet.}$$

By experiment 30, we have

$$Q = 138.1892 \text{ cubic feet per second,}$$
$$h = 12.903 \text{ feet,}$$
$$\sqrt{2g} = 8.0202 \text{ feet,}$$

consequently,

$$138.1892 = 7.68694 \times 8.0202 \sqrt{12.903}\; C,$$
$$\text{or } C = 0.624.$$

By rule 2, we have $H = 0.10\,D$:

$$\text{then } HD = 0.10\,D^2,$$
$$\text{and } Q = HDV' = 0.10\,D^2\,C\sqrt{2gh},$$
$$\text{or } Q = 0.5\,D^2\sqrt{h}.$$

Calling the weight of a cubic foot of water 62.33 pounds avoir. we have

$$P = \frac{0.75 \times 62.33}{550}\,Q\,h,$$
$$\text{or } P = 0.085\,Q\,h;$$

or, substituting the value of Q just found,

$$P = 0.0425\,D^2\,h\sqrt{h},$$

from which we may deduce

$$D = 4.85\sqrt{\frac{P}{h\sqrt{h}}}.$$

92. The number of buckets is, to a certain extent, arbitrary, and would usually be determined by practical considerations: some of the ideas to be kept in mind are the following.

The pressure on each bucket is less, as the number is greater; the greater number will therefore permit of the use of the thinner iron, which is important, in order to obtain the best results. The width of the crowns will be less for a greater number of buckets: a narrow crown appears to be favorable to the useful effect, when the gate is only partially raised. As the spaces between the buckets must be proportionally narrower for a larger number of buckets, the liability to become choked up, either with anchor ice, or other substances, is increased. The amount of power lost by the friction of the water against the surfaces of the buckets, will not be materially changed, as the total amount of rubbing surface on the buckets, will be nearly constant for the same diameter: there will be a little less on the crown, for the larger number. The cost of the wheel will probably increase with the number of buckets. The thickness and quality of the iron, or other metal intended to be used for the buckets, will sometimes be an element. In some waters, wrought iron is rapidly corroded.

The author is of opinion that a general rule cannot be given for the number of buckets; among the numerous turbines working satisfactorily in Lowell, there are examples in which the shortest distance between the buckets is as small as 0.75 inches, and in others as large as 2.75 inches.

As a guide in practice, to be controlled by particular circumstances, the following is proposed; to be limited to diameters of not less than two feet;

$$N = 3\,(D + 10).$$

Taking the nearest whole number for the value of N.

The Tremont Turbine is $8\frac{1}{2}$ feet in diameter, and, according to the proposed rule, should have fifty-five buckets, instead of forty-four. With fifty-five buckets, the crowns should have a width of 7.2 inches, instead of 9 inches; with the narrower width, it is probable that the useful effect, in proportion to the power expended, would have been a little greater when the gate was partially raised.

92. By the 3d rule, we have for the width of the crowns,

$$W = \frac{4\,D}{N};$$

and for the interior diameter of the wheel

$$d = D - \frac{8\,D}{N}.$$

By the 4th rule, d is also equal to the sum of the shortest distances between the guides, where the water leaves them.

93. The number n, of the guides, is, to a certain extent, arbitrary; the practice at Lowell has been, usually, to have from a half to three fourths of the number of the buckets; exactly half would probably be objectionable, as it would tend to produce pulsations, or vibrations.

94. The proper velocity to be given to the wheel, is an important consideration. Experiment 30, on the Tremont Turbine, gives the maximum coefficient of effect for that wheel; in that experiment the velocity of the interior circumference of the wheel, is 0.62645 of the velocity due to the fall acting upon the wheel. By reference to the other experiments with the gate fully raised, it will be seen, however, that the coefficient of effect varies only about two per cent. from the maximum, for any velocity of the interior circumference, between fifty per cent. and seventy per cent. of that due to the fall acting upon the wheel. By reference to the experiments in which the gate is only partially raised, it will be seen that the maximum corresponds to slower velocities; and as turbines,

to admit of being regulated in velocity for variable work, must, almost necessarily, be used with a gate not fully raised, it would appear proper to give them a velocity such, that they will give a good effect under these circumstances.

With this view, the following is extracted from the experiments in table II.

Number of the exp[redacted]t.	Height of the regulating gate in feet.	Ratio of the velocity of the interior circumference of the wheel to the velocity due the fall acting upon the wheel corresponding to the [redacted]mum coefficient of effect
80	11.43	0.62015
62	8.55	0.56511
78	5.65	0.56205
84	2.875	0.48490

By this table it would appear, that, as turbines are generally used, a velocity of the interior circumference of the wheel, of about fifty-six per cent. of that due to the fall acting upon the wheel, would be most suitable. By reference to the diagram at plate VI, it will be seen that, at this velocity when the gate is fully raised, the coefficient of effect will be within less than one per cent. of the maximum.

Other considerations, however, must usually be taken into account, in determining the velocity; the most frequent is the variation of the fall under which the wheel is intended to operate. If, for instance, it was required to establish a turbine of a given power, on a fall liable to be diminished to one half, by backwater, and, that the turbine should be of a capacity to give the requisite power at all times; in this case, the dimensions of the turbine must be determined for the smallest fall; but if it has assigned to it a velocity, to give the maximum effect at the smallest fall, it will evidently move too slow for the greatest fall; and this is the more objectionable, as, usually, when the fall is greatest, the quantity of water is the least, and it is of the most importance to obtain a good effect. It would then be usually, the best arrangement, to give the wheel a velocity corresponding to the maximum coefficient of effect, when the fall is the greatest. To assign this velocity, we must first find the proportional height of gate, when the fall is greatest; this may be determined approximately by aid of the experiments on the Tremont Turbine.

We have seen that $P = 0.085\, Q h$.

Now, if h is increased to $2h$, the velocity, and, consequently, the quantity of water discharged, will be increased in the proportion of $\sqrt{h}$ to $\sqrt{2h}$; that is to say, the quantity for the fall $2h$, will be $\sqrt{2}\, Q$

Calling P' the total power of the turbine on the double fall, we have

$$P' = 0.085 \sqrt{2}\, Q\, 2 h,$$

or

$$P' = 0.085 \times 2.8284\, Q h.$$

Thus, the total power of the turbine is increased 2.8284 times, by doubling the fall; on the double fall, therefore, in order to preserve the effective power uniform, the regulating gate must be shut down to a point that will give only $\frac{1}{2.8284}$ part of the total power of the turbine.

In experiment 15, the fall acting upon the wheel was 12.888 feet, and the total useful effect of the turbine was 85625.3 pounds raised one foot per second; $\frac{1}{2.8284}$ part of this is 30273.4 lbs; consequently, the same opening of gate that would give this last power, on a fall of 12.888 feet, would give a power of 85625.3 lbs. raised one foot per second, on a fall of 2 × 12.888 feet = 25.776 feet. To find this opening of gate, we must have recourse to some of the other experiments.

In experiment 78, the fall was 13.310 feet, the height of gate 5.65 inches, and the useful effect 58830.1 pounds. In experiment 83, the fall was 13.435 feet, the height of gate 2.875 inches, and the useful effect, 27310.9 pounds. Reducing both these useful effects to what they would have been, if the fall was 12.888 feet,—

the useful effect in experiment 73, $58830.1 \left(\frac{12.888}{13.310}\right)^{\frac{3}{2}} = 56054.5,$

" " " 83, $27310.9 \left(\frac{12.888}{13.435}\right)^{\frac{3}{2}} = 25660.1.$

By a comparison of these useful effects with the corresponding heights of gate, we find, by simple proportion of the differences, that a useful effect of 30273.4 pounds raised one foot high per second, would be given when the height of the regulating gate was 3.206 inches.

By another mode:—

$$\text{as } 25660.1 : 2.875 :: 30273.4 : 2.875 \times \frac{30273.4}{25660.1} = 3.392 \text{ inches,}$$

a little consideration will show, that the first mode must give too little, and the second, too much; taking a mean of the two results, we have for the height of the gate, giving $\frac{1}{2.8284}$ of the total power of the turbine, 3.344 inches. Referring to table II, we see that, with this height of gate, in order to obtain the best coefficient of useful effect, the velocity of the interior circumference of the wheel, should be about one half of that due to the fall acting upon the wheel; and by comparison of experiments 71 and 81, it will be seen that, with this height of gate, and with this velocity, the coefficient of useful effect must be near 0.50.

This example shows, in a strong light, the well-known defect of the turbine, viz, giving a diminished coefficient of useful effect, at times when it is important to obtain the best results. One remedy for this defect would be, to have a spare turbine, to be used when the fall is greatly diminished; this arrangement would permit the principal turbine to be made nearly of the dimensions required for the greatest fall. As at other heights of the water, economy of water is usually of less importance, the spare turbine might generally be of a cheaper construction.

95. *To lay out the curve of the buckets,* the author makes use of the following method.

Referring to plate III., figure 1, the number of buckets, N, having been determined by the preceding rules, set off the arc $gi = \frac{\pi D}{N}$.

Let $\omega = gh$, the shortest distance between the buckets;
$t =$ the thickness of the metal forming the buckets.

Make the arc $gk = 5\,\omega$. Draw the radius Ok, intersecting the interior circumference of the wheel at l; the point l will be the inner extremity of the bucket. Draw the directrix lm tangent to the inner circumference of the wheel. Draw the arc on, with the radius $\omega + t$, from i, as a centre; the other directrix, gp, must be found by trial, the required conditions being, that, when the line ml is revolved round to the position gl, the point m being constantly on the directrix gp, and another point at the distance $mg = rs$, from the extremity of the line describing the bucket, being constantly on the directrix ml, the curve described shall just touch the arc ns. A convenient line for a first approximation, may be drawn by making the angle $Ogp = 11°$. After determining the directrix according to the preceding method, if the angle Ogp should be greater than 12°, or less than 10°, the length of the arc gk should be changed, to bring the angle within these limits.

The curve $gss's''l$, described as above, is nearly the quarter of an ellipse, and would be precisely so, if the angle gml was a right angle; the curve may be readily described, mechanically, with an apparatus similar to the elliptic trammel; there is, however, no difficulty in drawing it by a series of points, as is sufficiently obvious.

96. The trace adopted by the author, for the corresponding guides, is as follows.

The number n having been determined, divide the circle, in which the extremities of the guides are found, into [illegible] equal parts, $v w$, $w v$, etc.

Put w' for the width between two adjoining guides,

and t' for the thickness of the metal forming the guides.

We have by rule 4, $w' = \frac{d}{n}$.

With [illegible] as a centre, and the radius $w' + t'$, draw the arc $y z$; and with x as a centre, and the radius $2(w' + t')$, draw the arc $a' b'$. Through v draw the portion of a circle $v c'$, touching the arcs $y z$ and $a' b'$; this will be the curve for the essential part of the guide. The remainder of the guide, $c' d'$, should be drawn tangent to the curve $c' v$; a convenient radius is one that would cause the curve $c' d'$, if continued, to pass through the centre O. This part of the guide might be dispensed with, except that it affords great support to the part $c' v$, and thus permits the use of much thinner iron than would be necessary, if the guide terminated at c', or near it.

97. Collecting together the foregoing formulas for proportioning turbines, which, it is understood, are to be limited to falls not exceeding forty feet, and to diameters not less than two feet; we have

for the horse-power,

$$P = 0.0425\, D^2 h \sqrt{h};$$

for the diameter,

$$D = 4.85 \sqrt{\frac{P}{h \sqrt{h}}};$$

for the quantity of water discharged per second,

$$Q = 0.5\, D^2 \sqrt{h};$$

for the velocity of the interior circumference of the wheel, when the fall is not very variable,

$$v = 0.53 \sqrt{2gh},$$

or,

$$v = 4.401 \sqrt{h};$$

for the height of the orifices of discharge,

$$H = 0.10\, D;$$

for the number of buckets,

$$N = 3(D + 10);$$

for the shortest distance between two adjacent buckets,

$$w = \frac{D}{N};$$

for the width of the crown occupied by the buckets,

$$W = \frac{4D}{N};$$

for the interior diameter of the wheel,

$$d = D - \frac{8D}{N};$$

for the number of guides,

$$n = 0.50\,N \text{ to } 0.75\,N;$$

for the shortest distance between two adjacent guides,

$$w' = \frac{d}{n}.$$

Table IV. has been computed by these formulas.

For falls greater than forty feet the height of the orifices in the circumference of the wheel, should be diminished; the foregoing formulas may, however, still be made use of; thus, supposing that for a high fall, it is determined to make the orifices three fourths of that given by the formula; divide the given power, or quantity of water to be used, by 0.75, and use the quotient in place of the true power, or quantity, in determining the dimensions of the turbine; no modification of the dimensions will be necessary, except that $\frac{1}{10}$ of the diameter of the turbine should be diminished to $\frac{1}{13}$ of the diameter, to give the height of the orifices in the circumference.

98. It is plain, from the method by which the preceding formulas have been obtained, that they cannot be considered as established, but should only be taken as guides in practical applications, until some more satisfactory are proposed, or the intricacies of the turbine have been more fully unravelled. The turbine has been an object of deep interest to many learned mathematicians, but, up to this time, the results of their investigations, so far as they have been published, have afforded but little aid to Hydraulic Engineers.

TABLE IV.

Table for Turbines of different diameters, operating on different falls, assuming that the useful effect is seventy-five per cent. of the power expended, also that the velocity of the interior circumference is fifty-six per cent. of the velocity due the fall; and also that the height between the crowns is $\frac{1}{10}$ of the outside diameter.

Fall in feet.	Outside diameter 2.000 feet Inside " 1.550 " Number of buckets 30			Outside diameter 3.000 feet Inside " 2.385 " Number of buckets 30			Outside diameter 4.000 feet Inside " 3.238 " Number of buckets 42			Outside diameter 5.000 feet Inside " 4.111 " Number of buckets 45			Outside diameter 6.000 feet Inside " 5.000 " Number of buckets 48		
	Quantity of water discharged in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.
5	4.47	1.90	123.9	10.06	4.28	80.4	17.88	7.60	59.2	27.95	11.88	46.7	40.25	17.11	38.1
6	4.90	2.50	135.1	11.02	5.62	88.1	19.60	9.99	64.9	30.62	15.61	51.1	44.09	22.49	42.0
7	5.29	3.15	145.9	11.91	7.08	95.2	21.17	12.59	70.1	33.07	19.68	55.2	47.62	28.34	45.4
8	5.66	3.85	156.0	12.73	8.66	101.7	22.63	15.39	74.9	35.35	24.04	59.0	50.91	34.62	48.5
9	6.00	4.59	165.1	13.50	10.33	107.9	24.00	18.36	79.5	37.50	28.69	62.6	54.00	41.31	51.5
10	6.32	5.38	174.4	14.23	12.10	113.7	25.30	21.50	83.8	39.53	33.60	66.0	56.92	48.38	54.2
11	6.63	6.20	182.9	14.92	13.95	119.3	26.53	24.81	87.9	41.46	38.76	69.2	59.70	55.82	56.5
12	6.93	7.07	191.0	15.59	15.90	124.6	27.71	28.27	91.8	43.30	44.17	72.3	62.36	63.60	59.1
13	7.21	7.97	198.8	16.23	17.93	129.7	28.81	31.87	95.5	45.07	49.80	75.2	64.90	71.72	61.9
14	7.48	8.90	206.3	16.84	20.04	134.6	29.93	35.62	99.1	46.77	55.66	78.1	67.35	80.15	64.2
15	7.75	9.88	213.5	17.42	22.22	139.3	30.98	39.50	102.6	48.41	61.72	80.8	69.71	88.88	66.1
16	8.00	10.88	220.5	18.00	24.48	143.9	32.00	43.52	106.0	50.00	68.00	83.5	72.00	97.92	68.6
17	8.25	11.92	227.3	18.55	26.80	148.3	32.99	47.64	109.2	51.54	74.47	86.0	74.22	107.21	70.7
18	8.49	12.98	234.0	19.09	29.21	152.6	33.94	51.93	112.4	53.09	81.14	88.5	76.37	116.84	72.8
19	8.72	14.08	240.3	19.61	31.68	156.8	34.87	56.32	115.5	54.49	87.99	90.9	78.46	126.71	74.8
20	8.94	15.21	247.6	20.12	34.21	160.9	35.76	60.82	118.5	55.90	95.08	93.3	80.50	136.84	76.7
21	9.17	16.36	252.7	20.62	36.81	164.8	36.66	65.4	121.4	57.28	102.25	95.6	82.49	147.24	78.0
22	9.38	17.51	258.6	21.11	39.47	168.7	37.52	70.17	124.2	58.63	109.64	97.9	84.49	157.88	80.5
23	9.59	18.75	264.1	21.58	42.19	172.5	38.37	75.01	127.0	59.95	117.20	100.1	86.32	168.76	82.3
24	9.80	19.95	270.1	22.04	44.97	176.2	39.17	79.95	129.8	61.24	124.92	102.2	88.18	179.88	83.4
25	10.00	21.25	275.7	22.50	47.81	179.8	40.00	85.00	132.4	62.50	132.81	104.3	90.00	191.25	85.8
26	10.20	22.54	281.1	22.95	50.70	183.4	40.79	90.15	135.1	63.74	140.86	106.4	91.78	202.84	87.5
27	10.39	23.85	286.5	23.38	53.66	186.9	41.57	95.40	137.6	64.95	149.06	108.4	93.54	214.65	8
28	10.58	25.19	291.8	23.81	56.67	190.8	42.33	100.75	140.2	66.14	157.42	110.4	95.25	226.69	90.8
29	10.77	26.55	298.4	24.23	59.73	193.7	43.08	106.20	142.6	67.31	165.93	112.4	96.91	238.91	92.4
30	10.95	27.93	302.6	24.65	62.85	197.0	43.82	111.71	145.1	68.46	174.59	114.3	98.59	251.41	93.3
31	11.14	29.31	307.0	25.05	66.02	200.3	44.54	117.97	147.5	69.60	183.39	116.2	100.22	264.08	95.5
32	11.31	30.77	311.9	25.46	69.24	203.5	45.25	123.06	149.8	70.71	192.31	118.0	101.82	276.96	97.0
33	11.49	32.23	316.7	25.85	72.51	206.6	45.93	128.91	152.2	71.81	201.42	119.9	103.40	290.04	98.5
34	11.66	33.70	321.5	26.24	75.88	209.7	46.65	134.81	154.5	72.89	210.64	121.7	104.96	303.33	100.0
35	11.83	35.20	326.2	26.62	79.20	212.8	47.33	140.80	156.7	73.95	220.00	123.4	106.49	316.81	101.5
36	12.00	36.72	330.8	27.00	82.62	215.8	48.00	146.88	158.8	75.00	229.50	125.2	108.00	330.48	102.2
37	12.17	38.26	335.4	27.37	86.09	218.8	48.66	153.04	161.1	76.08	239.18	126.9	109.49	344.31	104.3
38	12.33	39.82	339.9	27.74	89.60	221.7	49.32	159.29	163.3	77.05	248.89	128.6	110.96	358.40	105.7
39	12.49	41.40	344.3	28.10	93.16	224.6	49.96	165.62	165.4	78.06	258.78	130.3	112.41	372.61	107.1
40	12.65	43.01	348.7	28.46	96.77	227.5	50.60	172.03	167.5	79.06	268.79	132.0	113.84	387.06	108.5

TABLE IV. — Continued.

Fall in feet	Outside diameter 7.000 feet. Inside " 6.003 " Number of buckets 51			Outside diameter 8.000 feet. Inside " [illegible] " Number of buckets 54			Outside diameter 9.000 feet. Inside " 7.[illegible] " Number of buckets 57			Outside diameter 10.000 feet. Inside " 8.[illegible] " Number of buckets 60		
	Quantity of water discharged, in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged, in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged, in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.	Quantity of water discharged, in cubic feet per second.	Number of horse-power.	Number of revolutions per minute.
5	54.78	23.28	32.5	71.55	30.41	28.1	90.56	38.49	24.8	111.80	47.52	22.1
6	60.01	30.61	35.6	78.38	39.97	30.8	99.20	50.59	27.2	122.47	62.46	24.2
7	64.82	38.57	38.4	84.67	50.37	33.3	107.15	63.76	29.3	132.29	78.71	26.2
8	69.30	47.12	41.1	90.51	61.55	35.6	114.55	77.90	31.4	141.42	96.17	28.0
9	73.50	56.23	43.6	96.00	73.44	37.8	121.50	92.95	33.3	150.00	114.75	29.7
10	77.47	65.86	46.0	101.19	86.02	39.8	128.07	108.85	35.1	158.11	134.40	31.3
11	81.26	75.97	48.2	106.13	99.23	41.7	134.32	125.59	36.8	165.83	155.05	32.8
12	84.87	86.57	50.3	110.85	113.07	43.6	140.30	143.10	38.4	173.21	176.67	34.3
13	88.31	97.61	52.4	115.38	127.49	45.4	146.03	161.36	40.0	180.28	199.21	35.7
14	91.67	109.09	54.4	119.73	142.48	47.1	151.53	180.33	41.5	187.08	222.63	37.0
15	94.89	120.98	56.3	123.94	158.02	48.7	156.86	199.98	42.9	193.65	246.90	38.3
16	98.00	133.28	58.1	128.00	174.08	50.3	162.00	220.32	44.3	200.00	272.00	39.6
17	101.02	145.97	59.9	131.94	190.65	51.9	166.99	241.28	45.7	206.16	297.90	40.8
18	103.94	159.03	61.7	135.76	207.72	53.4	171.83	262.89	47.0	212.13	324.56	42.0
19	106.79	172.47	63.3	139.48	225.27	54.9	176.53	285.10	48.3	217.94	351.98	43.1
20	109.57	186.26	65.0	143.11	243.28	56.3	181.12	307.91	49.6	223.61	380.13	44.3
21	112.27	200.11	66.6	146.61	261.75	57.7	185.60	331.28	50.8	229.13	408.99	45.4
22	114.91	214.89	68.2	150.09	280.67	59.0	189.96	355.23	52.0	234.52	438.55	46.4
23	117.50	229.71	69.7	153.47	300.03	60.4	194.23	379.72	53.2	239.79	468.79	47.5
24	120.02	244.85	71.2	156.77	319.81	61.7	198.41	404.76	54.3	244.95	499.70	48.5
25	122.50	260.31	72.7	160.00	340.00	62.9	202.50	430.31	55.4	250.00	531.25	49.5
26	124.93	276.09	74.1	163.17	360.60	64.2	206.51	456.39	56.5	254.95	563.44	50.5
27	127.30	292.17	75.5	166.28	381.61	65.4	210.45	482.97	57.6	259.81	596.26	51.4
28	129.64	308.55	76.9	169.33	403.00	66.6	214.31	510.05	58.7	264.58	629.69	52.4
29	131.93	325.22	78.3	172.32	424.78	67.8	218.09	537.61	59.7	269.26	663.72	53.3
30	134.19	342.19	79.6	175.27	446.91	68.9	221.83	565.66	60.7	273.86	698.35	54.2
31	136.41	359.44	80.9	178.17	469.47	70.1	225.50	594.18	61.7	278.39	733.55	55.1
32	138.59	376.97	82.2	181.02	492.37	71.2	229.10	623.16	62.7	282.84	769.33	56.0
33	140.74	394.78	83.5	183.82	515.68	72.3	232.66	652.59	63.7	287.23	805.67	56.9
34	142.86	412.86	84.7	186.59	539.24	73.4	236.16	682.48	64.6	291.55	842.57	57.7
35	144.94	431.21	86.0	189.31	563.21	74.5	239.60	712.82	65.6	295.80	880.02	58.5
36	147.00	449.82	87.2	192.00	587.52	75.5	243.00	743.58	66.5	300.00	918.00	59.4
37	149.03	468.69	88.4	194.65	612.17	76.6	246.35	774.77	67.4	304.14	956.51	60.2
38	151.03	487.82	89.6	197.26	637.15	77.6	249.66	806.40	68.3	308.22	995.55	61.0
39	153.00	507.20	90.8	199.84	662.47	78.6	252.92	838.44	69.2	312.25	1035.11	61.8
40	154.95	526.83	91.9	202.39	688.12	79.6	256.15	870.80	70.1	316.23	1075.17	62.6

EXPERIMENTS ON A MODEL OF A CENTRE-VENT WATER-WHEEL, WITH STRAIGHT BUCKETS.

99. The author was led to this design by the consideration of the path of the water in passing through the wheel, according to the hypothesis in art 88. It is a wheel well suited for low falls, in which the water, over the wheel, may stand at its natural height, without requiring a vertical shaft of great length. Its simplicity and cheapness, combined with its other good qualities as a hydraulic motor, must recommend it for many such situations.

100. Plate VII, figure 1, is a general plan, and figure 2, a vertical section of the apparatus.

Figure 3 is a vertical section through the apertures in the guides and wheel; the guides and buckets are omitted to avoid confusion in the figure.

Figure 4 is a horizontal section of part of the guides and buckets, showing, also, the path of the water in experiment 3, according to the hypothesis in art. 88.

A is the wheel; the exterior diameter is $22\frac{7}{8}$ inches; the interior diameter is $19\frac{3}{8}$ inches; the height between the crowns, or *B C*, figure 3, is $2\frac{13}{16}$ inches; it carries thirty-six buckets, *E E*, figure 4, of steel, about $\frac{1}{32}$ of an inch in thickness, fastened to the wheel by means of the wooden cushions *F F*, figure 3; the upper cushions are screwed to the disc *D*, and the lower ones to the crown *G*. The disc *D* is of cast-iron, $\frac{5}{8}$ inch thick, with a suitable hub by which it is connected with the vertical shaft.

H H are guides of cast-iron, which direct the water into the wheel, and also support the plate *I*, which protects the wheel from pressure on its upper surface; the contraction of the streams entering the apertures between the guides, is diminished by the curved wooden garniture *K*; there are twenty-four guides. The mean shortest distance between the buckets at *a b*, figure 4, is 0.0389 feet; the mean shortest distance between the guides *c d*, figure 4, is 0.0437 feet; and the height of both is $2\frac{13}{16}$ inches = 0.2344 feet; we have, therefore, for the sum of the areas of the smallest sections between the guides,

$$0.0437 \times 0.2344 \times 24 = 0.24584 \text{ square feet.}$$

Similarly, the sum of the areas of the smallest sections between the buckets is

$$0.0339 \times 0.2344 \times 36 = 0.28600 \text{ square feet.}$$

The water is admitted into the forebay *L*, by the pipes *MM*; the diaphragm *N* is to diminish the agitation of the water.

101. The apparatus for gauging the water discharged by the wheel, consisted of the weir *O*, which had sharp edges; the depth on the weir was measured by a hook gauge, in the box *P*, which communicated, by a small aperture, with the surrounding water; the height of the water above the wheel was taken at a gauge in the box *Q*; this box was made sloping on one side, in order to permit a better view of the gauge. The zeros of both gauges were at the level of the top of the weir; consequently, the difference in the readings of the gauges gave at once the fall acting upon the wheel.

102. The apparatus for measuring the power, consisted of the Prony dynamometer *R*, attached to the upper part of the vertical shaft; the weights were applied by means of the bell crank *S*, figures 1, 2, and 5; the oscillations of the brake were diminished by the hydraulic regulator *T*, and the extent of the oscillations was limited by the stops *U U*. The speed of the wheel was obtained by means of a counter, driven by the worm *V*, attached to the top of the upright shaft; this was so arranged as to strike a bell once in fifty revolutions of the wheel.

In order to diminish the passive resistances, the weight, bearing upon the step *W*, was counterbalanced, in part, by other weights, one of which is represented at *y*, figure 2; these were attached to the brakes at the points *X X*, by vertical cords passing over pulleys; the weight, resting on the step when the wheel was immersed, and the dynamometer attached, was found to be 170 pounds; the counterbalance was 100 pounds, leaving 10 pounds bearing upon the step. The entire apparatus for measuring the power, was in equilibrium when there were no weights in the scale.

103. In all the experiments, except experiment 10, the brake was lubricated with oil; in experiment 10 water was used for this purpose; experiments 9 and 10 were identical in all other respects It was noticed in experiment 10 that the whole apparatus trembled very much; this must have consumed some power which is perceptible in the coefficients of effect. Experiment 9, in which oil was used, and in which the trembling of the apparatus was very slight, gives a coefficient of effect of 0.6922; while experiment 10, in which water was used to lubricate, the brake, and in which the trembling of the apparatus was very distinct, gave 0.6886 as the coefficient of effect.

104. All the apparatus was constructed with great care and precision; the surfaces of the cast-iron guides were ground smooth; and the cast-iron disc and lower crown of the wheel were turned true, and polished, in order to diminish, as much as possible, the resistance of the water to the motion of the wheel.

105. In table V., the quantity of water discharged has been calculated by the formula

$$Q = 3.33\,(l - 0.1\,n h)\,h^{\frac{3}{2}},$$

in which $Q =$ the quantity in cubic feet per second, $l =$ the length of the weir $= 3.003$ feet; $n =$ the number of end contractions $= 2$; $h =$ the depth upon the weir. The weights were obtained for the purpose from Mr. O. A. Richardson, the official sealer of weights and measures for the City of Lowell. The effective length of the lever of the dynamometer, was two feet. The temperature of the water was 63¼° Fahrenheit. Temperature of the air at 8^h, 35′ A. M., 68° Fahrenheit. The weight of a cubic foot of water is taken at 62.3128 pounds, which is deduced from table I.

If, in any experiment, the brake touched, even momentarily, either of the stops *U U*, it was rejected; with the use, however, of a regular and sufficient quantity of oil to lubricate the brake, and a properly constructed hydraulic regulator, there is seldom any difficulty from this cause, except at very low velocities.

TABLE V.

EXPERIMENTS ON A MODEL OF A CENTRE-VENT WATER-WHEEL.

1	2						3	4	5	6	7	8	9	10	11	12	13	14	15	16
No. of the experiment.	Time. June 19, 1847						Duration of the experiment, in seconds	Total number of revolutions of the wheel, during the experiment.	Weight in the scale, in pounds avoirdupois.	Useful effect, or the friction of the brake in pounds avoirdupois, raised one foot per second.	Total fall acting upon the wheel, in feet.	Depth of water on the weir, in feet.	Quantity of water passing the weir in cubic feet per second.	Total power of the water, in pounds raised one foot per second.	Ratio of the useful effect to the power expended.	Velocity due the fall acting on the wheel, in feet per second.	Velocity of the exterior circumference of the wheel, in feet per second.	Ratio of the velocity of the exterior circumference of the wheel, to the velocity due the fall acting on the wheel.	Quantity of water discharged by the wheel, reduced to a uniform fall of 2.5 feet, in cubic feet per second.	Ratio of the reduced quantity in the preceding column to the reduced quantity in experiment 3.
	Beginning of the experiment.			Ending of the experiment.																
	H.	min.	sec.	H.	min.	sec.														
1	9	41	9	9	51	23	614	700	16	229.22	2.5198	0.3649	2.1507	337.69	0.6788	12.731	6.827	0.5363	2.1422	0.9884
2	[illegible]	59	57	10	[illegible]	16½	559¼	750	14	235.83	2.4609	0.3662	2.1620	331.54	0.7113	12.581	8.028	0.6381	2.1791	1.0054
3	10	21	5	10	30	16½	551¼	850	12½	242.10	2.5017	0.3669	2.1681	337.98	0.7163	12.685	9.930	0.7976	2.1674	1.0000
4	10	52	10	10	52	26½	16½	still	25¾	0.	2.5160	0.3440	1.9714	309.07	0.	12.772	0.	0.	1.9651	0.9067
5	10	56	45	10	56	58	13	„	19¾	0.	2.5074	0.3426	1.9596	306.17	0.	12.700	0.	0.	1.9567	0.9078
6	11	4	0	11	4	26	26	„	19¾	0.	2.4405	0.3395	1.9334	294.02	0.	12.529	0.	0.	1.9564	0.9029
7	11	9	40	11	9	58½	18½	„	24⅛	0.	2.3735	0.3365	1.9082	282.23	0.	12.556	0.	0.	1.9584	0.9036
[illegible]	11	21	44	11	28	42	418	700	12	252.53	2.5977	0.3720	2.2127	358.17	0.7051	12.996	10.029	0.7759	2.1707	1.0015
9	11	45	32	11	52	16½	404½	700	11½	250.08	2.6059	0.3734	2.2250	361.29	0.6922	12.947	10.364	0.8005	2.1793	1.0055
10	3	3	57½	3	9	18½	321	550	11¼	247.61	2.5986	0.3729	2.2206	359.57	0.6886	12.929	10.261	0.7936	2.1781	1.0049
11	3	23	53	3	31	56½	483½	900	10	233.91	2.6001	0.3737	2.2276	360.84	0.6482	12.932	11.147	0.8620	2.1843	1.0078
12	3	45	1	3	50	11	310	700	[illegible]	170.25	2.6331	0.3750	2.2390	367.37	0.4634	13.014	13.523	1.0391	2.1817	1.0063
13	4	45	10½	4	49	50	279	750	0	0.	2.6287	0.3641	2.1437	351.15	0.	13.603	16.670	1.2359	2.0906	0.9648

106. In the foregoing table, experiments 4, 5, 6, and 7, were made with the wheel still; the brake was screwed up tight, and the pressure of the water upon the buckets, was measured by weights in the scale. In experiments 4 and 7, the weights were sufficient to balance the effect of the pressure of the water on the buckets, and also to overcome the friction of the apparatus; in other words, the weights were the least that would cause the scale to preponderate over the active and passive forces In experiments 5 and 6, the weights in the scale were the greatest that the pressure upon the buckets would raise, and overcome the friction of the apparatus; consequently, the force of the water acting upon the buckets, may be considered as balanced by the average of the weights in the fourth and fifth experiments, and, also, by the average in the sixth and seventh experiments.

To obtain the true weight that would balance the pressure, we must reduce the weights in the different experiments to what they would have been, if the fall acting upon the wheel had been constant

The following table shows the weights reduced to a uniform fall of 2.5 feet, obtained by simple proportion; thus, in the fourth experiment,

2.5160 : 25.75 .. 2.500 : 25.586.

The quantities discharged are also given for a uniform fall of 2.5 feet.

Number of experiment	Actual fall acting upon the wheel, in feet	Weight in scale by experiment in pounds	Weight reduced to a uniform fall of 2.5 feet	Quantity of water discharged reduced to a uniform fall of 2.5 feet, in cubic feet per second.
4	2.5160	25.750	25.586	1.9651
5	2.507	19.375	19.318	1.9567
6	2.4405	19.250	19.719	1.9568
7	2.3735	24.125	25.411	1.9584
Means			22.5085	1.9592

The mean reduced weight, when the weights preponderated, is 25.4985 pounds.
and when the pressure on the buckets preponderated, . 19.5185 "
Difference, ... 5.9800 pounds.

Half of this difference, or 2.99 pounds, may be considered as the measure of the passive resistances, or, rather, of the friction of the apparatus.

107. In experiment 13, the brake was entirely removed, and the wheel allowed to run without load; with the brake, the counterbalance was necessarily

removed, consequently the passive resistance arising from the friction of the step, was much greater than in the other experiments.

108. Fig. 6, plate VII, is a diagram representing the experiments; the abscissas represent the ratios of the velocities of the exterior circumference of the wheel, to the velocities due to the falls acting upon the wheel, as given in column 14, of table V.; the ordinates represent the ratios of the useful effects to the powers expended, as given in column 11; the points, representing experiments 12 and 13, are connected by a broken line, because the latter experiment is not strictly comparable with the others, in consequence of the removal of the counterbalance.

109. The following table contains the successive steps of the calculation for the ordinates of the path of the water in experiment 8, represented at figure 4, plate VII.; the operations are all similar to those explained in articles 83 and 119. The ordinates in column 10 are obtained by the formula

$$O = \frac{R' \omega A H}{Q''},$$

in which

O is the ordinate,

R' the corresponding value of the radius in column 1,

ω, the angular velocity $= \frac{850 \times 2\pi}{551.5} = 9.684$,

AH, the corresponding volume in column 9,

Q'', the mean quantity discharged by each aperture in the wheel $= \frac{2.1681}{36} = 0.06022$.

1 Values of R' and successive values of R', in inches	2 Areas in square inches of circles of radii in column 1	3 Areas in square inches of the corresponding rings	4 $\frac{1}{36}$ of the areas of the corresponding rings in square feet	5 Correction for thickness of the buckets, in square feet	6 Precise areas of the portions of rings, in square feet	7 Height of the part at ... in feet	8 Volumes of the parts of rings, in cubic feet	9 Volumes between ... and the successive values of R', in cubic feet	10 Ordinates in feet, measured on arcs of circles with radii in column 1
11.437	410.930								
11.000	380.183	30.808	0.005942	0.000262	0.005680	0.2811	0.001331	0.001331	0.1902
10.500	346.361	33.772	0.006515	0.000350	0.006165	"	0.001445	0.002776	0.3906
10.250	330.064	16.297	0.003144	0.000198	0.002946	"	0.000631	0.003407	0.4762
10.000	314.159	15.905	0.003068	0.000228	0.002840	"	0.000660	0.004067	0.5548
9.875	306.354	7.805	0.001500	0.000150	0.001350	"	0.000316	0.004410	0.5887
9.750	298.618	7.706	0.001486	0.000175	0.001311	"	0.000307	0.004750	0.6211

EXPERIMENTS ON THE POWER OF A CENTRE-VENT WATER-WHEEL, AT THE BOOTT COTTON-MILLS IN LOWELL, MASSACHUSETTS.

110. This wheel is one of a pair constructed from the designs of the author by the Lowell Machine Shop, for the Boott Cotton-Mills, in 1849. During a considerable portion of the year, the fall, on which these wheels operate, is about nineteen feet; with this fall, and with the regulating gates raised to the full height, they each furnish an effective power of about 230 horse-power.

A patent for the term of fourteen years was issued, July 26, 1838, by the Government of the United States of America, to Samuel B. Howd, of Geneva, in the State of New York, for a water-wheel resembling, in some respects, the wheels at the Boott Cotton-Mills.* Under this patent, a large number of wheels have been constructed, and a great many of them are now running in different parts of the country; they are known in some places as the *Howd wheel*, in others as the *United States wheel;* they have uniformly been constructed in a very simple and cheap manner, in order to meet the demands of a numerous class of millers and manufacturers, who must have cheap wheels if they have any.

111. Figures 3 and 4, plate IX., are a plan and vertical section of one of the Howd wheels, constructed by the owners of the patent right for a portion of New England. *A*, the wooden guides by which the water is directed on to the buckets; *B*, buckets of cast-iron, fastened to the upper and lower crowns of the wheel, by bolts, the upper crown is connected with the vertical shaft *E*, by the arms *C*. *D*, the regulating gate, placed *outside* of the guides; this is made of wood; the apparatus by which it is moved is not represented; it is a simple arrangement of levers. The upright shaft *E* runs on a step at the bottom This wheel is usually placed in the bottom of a rectangular forebay, which, in high falls, may be closed at the top, so as to avoid the necessity of using a vertical shaft of great length. The peculiarly shaped projections on one side of the buckets, it is said, increase the efficiency of the wheel, by diminishing the

---

* A wheel similar, in its essential features, was proposed in France, in 1826, by *Poncelet*.

waste of water; it is possible that some such effect may be produced by them. The author is not aware that any exact experiments have been made on the power of these wheels, from their form and construction, however, it is plain that they cannot be classed among those using water with very great economy. In the design for the Boott wheel, the author has so modified the form and arrangement of the whole, as to produce a wheel essentially different from the Howd wheel, as above described, although it may, possibly, be technically covered by the patent for that wheel.

112 Figures 1 and 2, plate VIII, are a vertical section, and a plan of the Boott centre-vent wheel, showing, also, the apparatus used in the experiments. *A*, the lower end of a pipe, about one hundred and thirty feet long, and eight feet in diameter, by which the water is conducted into the forebay *B*; this pipe is constructed of plate iron, three eighths of an inch in thickness, riveted together in the usual manner of making steam-boilers. For local reasons, the top of the forebay *B* is closed, so as to prevent the water from rising to its natural level, by about six or seven feet. *C*, the surface of the water in the Merrimack River, represented at about its medium height during the experiments. *D*, the wheel; *E*, the guides; *F*, the regulating gate, the apparatus for moving which, is not represented; *G*, the disc, which relieves the wheel from the vertical pressure of the water, and which also supports the lower bearing of the vertical shaft. The leather packing of the regulating gate *F*, slides against the circumference of the disc, which is turned smooth and cylindrical for that purpose, and the disc itself is supported by means of four brackets, two of which are represented at *HH* by the columns *II* The vertical shaft *K* is of wrought iron, and it passes through the stuffing box *L*, and is supported by the box *M*, which has a series of recesses lined with babbit metal, fitted to receive a corresponding series of projections in the vertical shaft The wheel, the vertical shaft, and the bevel gear usually on the latter, have a total weight of about 15,200 pounds; the bearing surface in the box *M* is about 331 square inches, consequently, the weight, per square inch, of bearing surface, is about 46 pounds

Figures 3 and 4, plate VIII., represent the wheel and guides on a larger scale. The buckets and guides are equal in number, there being forty of each; the buckets are of plate iron, $\frac{1}{4}$ of an inch in thickness; the guides are of the same material, $\frac{3}{16}$ of an inch in thickness. The following dimensions were taken after the parts were finished:—

Mean shortest distance between adjacent buckets, or *a b* figure 4, ... 0.1384 feet.

Mean height between the crowns, at the inner extremities of the buckets, or *cd*, figure 3,	1.2300	feet
Mean height between the crowns, at the outer extremities of the buckets, or *ef*, figure 3,	0.9990	“
Mean shortest distance between the adjacent guides, or *gh*, figure 4,	0.1467	“
Mean height of the orifices between the guides, or *ik*, figure 3,	1.0066	“
Diameter of the wheel at the outside of the buckets,	9.338	“
Diameter of the wheel at the inside of the buckets,	7.987	“

113. Several of the peculiar features of this design are covered by patents issued by the Government of the United States to U. A. Boyden. His patents cover the arrangement of the regulating gate, by placing it between the guides and the wheel, and having it detached from the garniture; making the height between the crowns of the wheel greater where the water is discharged, than where it enters; they also cover the self-adjusting apparatus on which the box *M* is supported.

114. Returning to figures 1 and 2, plate VIII, *N* is the friction pulley of the dynamometer, which is attached to the part of the shaft intended to receive the hub of the bevel gear, for the transmission of the power; *O*, the brake of maple wood; *P*, the bell crank, and *Q*, the hydraulic regulator; the friction pulley and the brake were subsequently used in the experiments on the Tremont Turbine, in the account of which they are more particularly described, (see arts. 37 and 38) *R*, the weir at which the water discharged by the wheel was gauged; *S*, a grating for the purpose of equalizing the flow of the water towards the weir; *T*, the gauge box in which the depths on the weir were observed. The communication between the water inside the box, and that surrounding it, was maintained by means of an aperture in the bottom of the box, (which extended 1.06 feet below the top of the weir,) and which was 4.12 feet from the weir. It may be thought, at first sight, that the depths on the weir were taken so near it, as to be affected by the curvature in the surface, caused by the discharge over the weir, but the experiments at the Lower Locks, (art. 173,) prove, conclusively, that when the communication between the water inside the box, and that outside of it, is maintained, by means of a pipe opening near the bottom of the canal, the depths are not affected in any appreciable degree, by the curvature in the surface. If any such effect was produced in this case, it must have been very slight *U* and *V* are the gauge boxes at which the heights of the water, below and above the wheel, were observed, in order to

obtain the fall acting upon the wheel. The velocity of the wheel was obtained by means of the counter *W*. The apparatus for lubricating the brake is not represented on the plate; in some of the experiments, water was used, and in others, linseed oil.

The experiments were made according to the method of continuous observations, which has been sufficiently described in the account of the experiments on the Tremont Turbine.

115. The experiments on the Boott centre-vent water-wheel, are given in detail in table VI, which will be intelligible, without much further explanation than is contained in the respective headings of the several columns.

116. COLUMN 10. *Useful effect, = the friction of the brake, in pounds avoirdupois raised one foot per second.* The brake was connected with the vertical arm of the bell crank, by a link, which was horizontal when the brake was in its normal position. When in this position, the length of a perpendicular, from the centre of the vertical shaft, to the line joining the points of the brake and bell crank to which the link was attached, was 9.743 feet; the effective length of the vertical arm of the bell crank, was 4.5 feet, and of the horizontal arm to which the scale was attached, 5 feet; consequently, the effective length of the brake was

$$\frac{9.743 \times 5}{4.5} = 10.826 \text{ feet.}$$

117. COLUMN 15. *Quantity of water passing the wheel, in cubic feet per second.* This quantity was gauged at the weir. The length of the weir was 13.908 feet; the width of the raceway on the upstream side of the weir, was 17 feet; the crest of the weir was 11.14 feet above the bottom of the raceway. The quantity has been computed by the formula

$$Q = 3.33\,(l - 0.1\,n\,h)\,h^{\frac{3}{2}},$$

determined from the experiments made, in 1852, at the Lower Locks. (See art. 258.) In this formula

Q = the quantity in cubic feet per second.
l = the length of the weir = 13.908 feet.
n = the number of end contractions = 2.
h = the depth on the weir, given in column 14.

TABLE

EXPERIMENTS ON THE BOOT

1	2	3	4	5						6	7	8	9	10
No. of the experiment.	DATE, 1849.	Temperature of the water in degrees of Fahrenheit's thermometer.	Height of the gauge in inches.	TIME						Duration of the experiment in seconds.	Total number of revolutions of the wheel during the experiment.	Number of revolutions of the wheel per second.	Weight in the scale, in pounds avoirdupois.	Useful effect, or the friction of the brake, in pounds avoirdupois, raised one foot per second.
				Beginning of the experiment.			Ending of the experiment.							
				H.	min.	sec.	H.	min.	sec.					
1	October 17, A.M.	54	3	10	13	10	10	17	32	253	150	0.59289	575.50	23211.8
2	" " "	"	"	11	30	8.5	11	36	42	398.5	350	0.87829	202.00	12073.5
3	" " "	"	"	11	40	11	11	51	15	484	350	0.72314	407.25	20032.3
4	" 20, "	49.5	"	11	2	37	11	19	7	1010	550	0.54455	600.00	22117.2
5	" " "	"	"	11	33	41	11	45	22	701	450	0.64194	504.44	22030.5
6	" " "	"	"	11	59	21	0	6	49	445	200	0.44944	720.50	22026.8
7	November 5, P.M.	44	"	4	14	47	4	21	20	393	100	0.25445	931.87	16129.1
8	" 7, A.M.	45	"	9	40	10	9	48	32	493	500	1.01420	0.	0.
9	October 17, P.M.	53	6	2	31	15.5	2	41	59	643.5	650	1.01010	331.06	22959.0
10	" " "	"	"	2	5	6	2	[illegible]	[illegible]	573	550	0.95986	441.22	28807.9
11	" " "	"	"	3	12	6	3	21	56	590	550	0.93220	501.72	31811.1
12	" " "	"	"	4	7	9	4	15	22.5	493.5	450	0.91185	502.59	31176.0
13	" " "	"	"	4	50	36	5	4	13.5	817.5	700	0.85627	656.59	38248.0
14	" 20, "	50	"	2	4	10	2	14	58	648	450	0.69444	955.50	45135.3
15	" " "	"	"	2	41	41	2	52	50	669	400	0.59791	1110.94	46102.8
16	November 7, A.M.	45	"	9	29	27	9	37	57	510	600	1.17647	0.	0.
17	October 20, P.M.	50	9	3	20	41	3	27	28	407	450	1.10565	263.00	19779.8
18	" " "	"	"	3	31	18	3	38	51	453	450	0.99338	531.75	35931.0
19	" " "	"	"	3	36	1	3	4	8	[illegible]	400	0.90000	786.75	48912.7
20	" " "	"	"	[illegible]	45	5	3	51	10.5	547.5	450	0.82193	1001.47	56195.7
21	" " "	"	"	3	55	14	4	1	51.5	699.5	550	0.78628	1107.37	59226.1
22	" " "	"	"	4	21	14	4	30	10	536	400	0.74627	1205.00	61168.8
23	" " "	"	"	4	31	19	4	40	31	552	400	0.72464	1259.16	62065.1
24	" " "	"	"	4	41	42	4	51	4.5	562.5	400	0.71111	1297.11	62752.2
25	" " "	"	"	4	54	35	5	4	7.5	572.5	400	0.69869	1329.78	63199.3
26	November 7, A.M.	45	"	9	19	24	9	27	22.5	478.5	600	1.25392	0.	0.
27	November 5, A.M.	44	12	9	4	34.5	9	13	58.5	564	400	0.70922	1551.22	74970.8
28*	" " "	"	"	9	15	10	9	21	7.5	357.5	250	0.69930	1584.00	75817.2
29	" " "	"	"	9	33	15	9	39	23	368	250	0.67935	1619.01	74580.9
30	" " "	"	"	9	40	37	9	48	3.5	446.5	300	0.67189	1614.97	751[illegible]
31	" " "	"	"	10	0	8	10	7	37.5	449.5	300	0.66741	1675.06	75208.3
32	" " "	"	"	10	8	54.5	10	16	37	462.5	300	0.64865	1715.17	752[illegible]
33	" " "	"	"	10	32	31	10	41	41	550	350	0.63636	1785.91	751[illegible]
34	" " "	"	"	10	48	0	10	56	0.5	480.5	300	0.62435	1768.11	75103.3
35	" " "	"	"	11	1	58	11	10	2	484	300	0.61983	1802.06	75202.0
36	" " "	"	"	11	11	24	11	18	26	422	250	0.59242	1846.19	73[illegible]
37	" 6, P.M.	"	"	3	31	12	3	35	20	248	0	0.	3155.31	0.
38	" " "	"	"	3	40	16	3	42	22	126	0	0.	2797.27	0.
39	" 7, A.M.	45	"	9	0	57	9	18	5	428	550	1.28505	0.	0.

In experiments Nos. 8, 16, 26, and 39, the brake was removed.

In experiment No. 37, the weight preponderated. In No. 38, the wheel preponderated (art. 77.)

VI.

CENTRE-VENT WATER-WHEEL

No. of the experiment	11 Height of the water [illegible] Feet	12 Height of the [illegible] taken in the wheel-pit Feet	13 Total fall acting upon the wheel. Feet	14 Depth of water on the weir. Feet	15 Quantity of water passing the wheel, in cubic feet, per second	16 Total power of the water, in pounds avoirdupois, raised one foot per second	17 Ratio of the useful effect to the power expended	18 Velocity due to the fall acting upon the wheel, in feet per second	19 Velocity of the exterior circumference of the wheel, in feet per second	20 Ratio of the velocity of the exterior circumference of the wheel, to the velocity due to the fall acting upon the wheel
1	16.013	1.410	14.603	1.2964	67.582	61493.4	0.37747	30.648	17.393	0.56750
2	16.033	1.361	14.672	1.2619	64.887	59320.4	0.20988	30.721	25.766	0.83871
3	15.955	1.387	14.568	1.2821	66.482	60347.0	0.33195	30.612	21.214	0.69301
4	15.558	1.400	14.158	1.2845	66.614	58821.6	0.38161	30.178	15.975	0.52937
5	15.607	1.410	14.197	1.2899	67.029	59351.7	0.38129	30.219	14.647	0.48470
6	15.568	1.420	14.148	1.2881	66.880	59002.2	0.37832	30.162	13.185	0.43714
7	15.604	1.360	14.244	1.2948	67.368	59858.1	0.20046	30.269	7.465	0.24661
8	15.573	1.273	14.300	1.2115	61.083	54480.4	0.	30.329	29.753	0.98101
9	15.956	1.668	14.288	1.5145	84.998	75782.8	0.30808	30.316	29.633	0.97740
10	15.930	1.701	14.228	1.5308	86.455	76608.0	0.37604	30.250	28.159	0.93086
11	15.914	1.717	14.197	1.5395	87.080	77093.2	0.41267	30.219	27.347	0.90496
12	15.928	1.730	14.198	1.5407	87.685	77607.2	0.44424	30.215	26.429	0.87470
13	15.944	1.750	14.194	1.5539	88.285	78113.8	0.48030	30.216	25.120	0.83134
14	15.581	1.803	13.778	1.5762	90.166	77480.4	0.53254	29.770	20.372	0.68433
15	15.481	1.875	13.606	1.5948	91.697	77812.1	0.50634	29.584	17.540	0.59291
16	15.451	1.506	13.945	1.4180	77.112	67076.7	0.	29.950	34.518	1.15287
17	15.408	1.890	13.518	1.6118	95.762	80736.8	0.24400	29.488	32.436	1.09997
18	15.328	1.950	13.378	1.6734	98.490	82145.2	0.43741	29.329	29.142	0.99362
19	15.352	1.983	13.369	1.6955	100.418	83728.2	0.57582	29.325	26.429	0.90125
20	15.413	2.017	13.396	1.7184	102.421	85571.4	0.65671	29.354	24.200	0.82442
21	15.426	2.047	13.379	1.7230	102.825	85800.0	0.69020	29.336	23.066	0.78629
22	15.418	2.076	13.342	1.7308	103.517	86138.0	0.71018	29.295	21.898	0.74781
23	15.424	2.102	13.322	1.7337	103.763	86218.8	0.71986	29.273	21.258	0.72620
24	15.465	2.131	13.334	1.7328	103.680	86229.8	0.72774	29.286	20.861	0.71232
25	15.464	2.160	13.304	1.7380	104.229	86483.7	0.73077	29.253	20.497	0.70067
26	15.417	1.715	13.702	1.5381	92.013	78048.0	0.	29.688	36.785	1.23907
27	15.398	1.998	13.400	1.8316	112.525	94057.5	0.79710	29.359	20.806	0.70868
28	15.[illegible]	2.009	13.431	1.8367	112.987	94662.2	0.79596	29.393	20.515	0.69796
29	15.321	1.990	13.331	1.8320	112.562	93609.[illegible]	0.79677	29.283	19.929	0.68058
30	15.369	1.991	13.378	1.8368	112.993	94296.1	0.79699	29.335	19.711	0.67193
31	15.367	1.981	13.386	1.8377	113.071	94415.2	0.79657	29.343	19.304	0.65790
32	15.369	1.986	13.383	1.8387	113.161	94171.1	0.79653	29.340	19.029	0.64856
33	15.336	1.980	13.356	1.8379	113.090	94219.0	0.79758	29.311	18.668	0.63692
34	15.362	1.981	13.381	1.8448	113.673	94881.9	0.79154	29.338	18.331	0.62481
35	15.385	1.980	13.405	1.8511	114.293	95571.2	0.78687	29.364	17.998	0.61291
36	15.292	1.971	13.321	1.8476	113.969	94703.1	0.78132	29.272	17.870	0.59871
37	15.442	1.905	13.537	1.8087	110.454	93270.1	0.	29.508	0.	0.
38	15.477	1.902	13.575	1.8072	110.325	93422.4	0.	29.550	0.	0.
39	15.415	1.819	13.596	1.6884	99.795	84635.0	0.	29.573	37.698	1.27477

118. The results of the experiments in table VI, are represented by a system of coördinates at figure 1, plate IX.; — the relative velocities, given in column 20, are taken for the abscissas, and the corresponding ratios of the useful effects to the powers expended, given in column 17, are taken for the ordinates. The numbers on the figure refer to the experiments in table VI, which the several points represent; — the points not numbered represent some experiments not reported, in consequence of an imperfection in the gauge of the quantity of water discharged, owing to a defective arrangement of the grating. These experiments have been corrected by a comparison with those that are reported; notwithstanding this correction, however, they ought not to be considered as of equal value with those reported in table VI. In the figure, the points representing the latter experiments, are connected by full lines, the points representing the experiments considered imperfect, are connected by broken lines. The line *AB* represents the experiments reported, that were made with the regulating gate fully raised; the line *CD*, the experiments with the gate raised three quarters of its full height; *EF*, the experiments with the gate raised a half, and *GH*, the experiments with the gate raised one quarter of its full height. It will be seen that the maximum coefficient of effect, with the gate fully raised, is given, when the outside of the wheel is moving with a velocity equal to about sixty-seven per cent. of that due to the fall acting upon the wheel, at which velocity, the useful effect is very nearly eighty per cent. of the total power of the water. The coefficient of effect diminishes rapidly as the regulating gate is lowered, and the maximum is also found at a slower speed; thus, when the gate is raised three inches, or one quarter of its full height, the maximum coefficient of effect is thirty-eight per cent. of the power expended; which is given when the outside of the wheel is moving with a velocity about one half of that due to the fall acting upon the wheel.

119. *AB CD*, figure 2, plate IX., represents the path of the water as it passed through one of the apertures of the wheel, in experiment 80, according to the hypothesis in art. 83; the steps in the calculation for which, are given in table VII. In the formula

$$O = \frac{R' w A H}{Q'},$$

we have for this case,

O = the ordinate measured on the arc of a circle the radius of which is R'; its several values are given in column 10.

R' = the distance from the centre of the wheel for which the ordinate is computed;—its several values are given in inches, in column 1;—to compute the value of O in feet, R' must be taken in feet.

ω = the angular velocity. In experiment 30, the velocity of the outside of the wheel was 19.711 feet per second, and the radius of the outside of the wheel is 4.669 feet, consequently,

$$\omega = \frac{19.711}{4.669} = 4.2217.$$

AH = the volume of that part of the space between two adjacent buckets, included between the outside of the wheel and the radius R';—its several values are given in column 9.

Q'' = the quantity of water discharged, per second, by each orifice in the wheel. In experiment 30, we have, by table VI, the total quantity discharged = 112.996 cubic feet per second, and as there are forty orifices, we have

$$Q'' = \frac{112.996}{40} = 2.8249.$$

In figure 2, plate IX., the buckets and guides are drawn to a scale one fourth the full size;—the radius of the arc $AB = R = 56.028$ inches. To find the limit of the stream on the side BC, the arcs IF, KH, etc., NC, are drawn with the radii 55 inches, 54 inches, etc., 47.922 inches;—the arcs EF, GH, etc., OC, being taken from column 10, equal to 0.415 feet, 0.796 feet, etc., 2.748 feet; the points B, F, H, etc., C, being connected by suitable lines, determine the limit of the stream on that side. The limit of the stream on the other side is found by making the arcs $FL = EI$, $HM = GK$, etc., $CD = ON$;—the points A, L, M, etc., D, being connected by suitable lines, determine the limit of the stream on that side.

By an examination of figure 2, it will be seen, that the section of the stream just after it has entered the wheel, is sensibly greater than the section of the stream as it leaves the guides, and that, consequently, if the stream flowed according to the hypothesis, there must have been a sudden change in the velocity of the water, causing a shock, which, according to the common theory, implies a loss of power. This indicates a defect in the design; nevertheless, the success attending this first essay, on a large scale, of a centre-vent water-wheel, in which due regard has been paid to accuracy of construction and perfection of workmanship, guided by such light as the present imperfect theories can afford, ought to encourage us to hope, that, when it has received the same degree of attention as the turbine, it will not be much behind, that celebrated motor, in its economical use of water.

TABLE VII.

1	2	3	4	5	6	7	8	9	10
Values of R and successive values of R', in inches	Areas in square inches of circles of the radii in the preceding column.	Areas in square inches of the rings	$\frac{1}{40}$ of the areas of the rings in preceding column, in square feet	Correction for the thickness of the buckets, in square feet	Corrected areas of the partial rings, in square feet	Mean [illegible] in feet	Volumes of the partial rings, in cubic feet	Volumes between R and the successive values of R', in cubic feet	[illegible]
56.028	9861.890								
55.000	9503.318	358.572	0.06225	0.00168	0.06057	1.001	0.06064	0.06064	0.115
54.000	9160.881	342.434	0.05945	0.00210	0.05735	1.008	0.0578	0.1184	0.7[illegible]
53.000	8824.734	336.150	0.05896	0.00227	0.05669	1.021	0.05727	0.17571	1.100
52.000	8494.866	329.868	0.05727	0.00232	0.05495	1.042	0.05695	0.23266	1.507
51.000	8171.282	323.584	0.05618	0.00304	0.05314	1.070	0.05686	0.28952	1.831
50.000	7853.982	317.300	0.05509	0.00386	0.05123	1.105	0.05661	0.34613	2.155
49.000	7542.961	311.018	0.05400	0.00561	0.04839	1.147	0.05550	0.40163	2.451
48.750	7466.191	76.770	0.01333	0.00168	0.01165	1.177	0.01371	0.41534	2.522
48.500	7389.811	76.380	0.01326	0.00181	0.01145	1.190	0.01363	0.42897	2.591
48.250	7313.824	75.987	0.01319	0.00202	0.01117	1.201	0.01345	0.44242	2.659
47.922	7214.723	99.101	0.01721	0.00252	0.01469	1.221	0.01794	0.46036	2.718

PART II.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, AND IN SHORT RECTANGULAR CANALS

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS

120. The laws governing the flow of water over weirs, have received the attention of several distinguished engineers and men of science, among whom may be named Smeaton and Brindley in England; Du Buat, Navier, D'Aubuisson, Castel, Poncelet, Lesbros, and Boileau, in France; and Eytelwein and Weisbach in Germany A great number of experiments have been made and recorded; the earlier ones rude and imperfect; the later ones, particularly those by Poncelet, Lesbros, and Boileau, with a perfection of apparatus previously unknown.

There has been in this branch of hydraulics, as well as in others, a steady advance with the accumulation of experiments and the improvement of the means of observation; the result, however, of these numerous labors, is far from satisfactory to the practical engineer. On a careful review of all that has been done, he finds that the rules given for his use, are founded on the single natural law governing the velocity of fluids, known [illegible] the theorem of Torricelli; omitting, in consequence of the extreme complexity of the subject, all consideration of many other circumstances, which, it is well known, materially affect the flow of water through orifices. He finds also that it has been attempted to correct the theoretical expression thus found, by coefficients obtained by comparing the results derived from it, with those furnished by experiment; but when he comes to investigate these experiments, even after rejecting all excepting those made with the greatest care, and with apparatus capable of insuring the greatest precision, he finds such discordances in the resulting coefficients, that he loses all hope of arriving at correct results when he applies them on the great scale. They will undoubtedly furnish sufficiently accurate results, if the apparatus used is a reproduction, both in form and dimension, of that used in the experiments; but this is seldom attainable, the experiments having been made on such a minute scale. Boileau,* in discussing the various formulas that have been proposed, points out many of their defects, and has himself proposed a new one, coupled, however, with some special conditions in the form of the weir, and the mode of taking the depth upon the sill.

No correct formula for the discharge of water over weirs, founded upon natural laws, and including the secondary effects of these laws, being known, we must rely entirely upon experiments, taking due care in the application of any formula deduced from thence, not to depart too far from the limits of the experiments on which it is founded.

Engineers have generally agreed that the most convenient form of weir for gauging streams of water, is one which is cut in a vertical plane side of a reservoir, the sill being horizontal, the sides vertical, and the contraction complete. In order that the contraction may be complete, the sill and sides of the weir must be so far removed from the bottom and lateral sides of the reservoir, that they may produce no more effect upon the discharge, than if they were removed a distance indefinitely great; also, the aperture must be effectively the same, as if cut in a plate having no sensible thickness. The condition relating to the distance of the bottom and sides of the reservoir, can seldom be strictly complied with, when gauging large streams of water; it is found, however, that, when the sill is at a height above the bottom of the reservoir not less than twice the height of the water above the sill, and the sides are removed a distance at least equal to the height above the sill, a correction free from serious error can usually be made for the effect of the velocity of the water approaching the weir. The condition that the aperture shall be effectively the same as if cut in a plate having no sensible thickness, is usually more easily complied with. The effect of the contraction is such, that the water has a strong tendency to leave the bottom and sides of the aperture for a certain distance, and to touch the aperture only at the upstream edge; if, however, the thickness of the plank or other material, exceeds a certain amount, (depending upon the depth flowing over,) the water will follow the top of the plank; in this case, all that is requisite is, to cut away the downstream side of the weir at an angle of, say, forty-five or sixty degrees with the horizontal; leaving horizontal, only a small part of the thickness of the sill. It is essential, however, that the corners of the sill and sides of the weir presented to the stream, should be full and sharp, and not rounded or bevelled in any degree.

---

* *Jaugeage des cours d'eau a faible ou a moyenne section* by M. P. Boileau (Paris: 1850); [illegible] *Journal de l'Ecole Polytechnique*, No xxxiii.

121. Two modes present themselves for studying, experimentally, the laws governing the discharge of water over weirs. *First*, that which has been uniformly adopted heretofore, namely, to obtain by direct measurement the quantity of water discharged in a given time, through an aperture of known dimensions; this is evidently the only mode of resolving the question completely. To perform the experiments, however, upon a scale of magnitude corresponding to the ordinary practical applications, usually requires an apparatus of great cost, and such as is beyond the reach of most experimenters. The great difficulty is, to obtain a suitable basin, in which to make the direct measurement of the quantity discharged by the weir.

The *second* mode dispenses with a direct measurement of the quantity. If we have two weirs of the same form, but of different lengths, and we know that the quantities of water discharged by them, in certain circumstances, are equal; knowing also the depth upon the sill of each weir, we have the data for an equation by which one unknown quantity may be determined. Neither the coefficient of contraction, nor the absolute discharge can, however, be obtained by such an equation.

122. The discharge over weirs is commonly assumed to vary as the square root of the third power of the depth; let us suppose it to be unknown, and equal to a.

Suppose also l the length, and h the depth, on one of the weirs; and l' and h' the corresponding dimensions for the other weir; C, a constant coefficient; Q, the quantity which, by hypothesis, is the same for both weirs. Assuming, according to the common formula, that the quantity is proportional to the length of the weir, we have

$$Q = Clh^a;$$
$$Q = Cl'h'^a;$$

consequently,

$$Clh^a = Cl'h'^a;$$
$$\left(\frac{h}{h'}\right)^a = \frac{l'}{l};$$

taking the logarithms, we have

$$a\,(\text{Log } h - \text{Log. } h') = \text{Log } l' - \text{Log. } l;$$

therefore,

$$a = \frac{\operatorname{Log} l' - \operatorname{Log} l}{\operatorname{Log} h - \operatorname{Log} h'}.$$

We can thus, by means of two experiments, determine the power of the depth which will lead to identical quantities in the computed discharge of the two weirs.

123. It is assumed in the above equations, that the quantity discharged by a weir is directly proportioned to its length; this, in weirs having complete contraction, is, however, known not to be true, in consequence of the contraction which takes place at the ends of the weir. This contraction diminishes the discharge. When the weir is of considerable length in proportion to the depth of the water flowing over, this diminution is evidently a constant quantity, whatever may be the length, provided the depth is the same; we may, therefore, assume that the end contraction effectively diminishes the length of such weirs, by a quantity depending only upon the depth upon the weir. It is evident that the amount of this diminution must increase with the depth; we are unable, however, in the present state of the science, to discover the law of its variation; but experiment has proved that it is very nearly in direct proportion to the depth As it is of great importance, in practical applications, to have the formula as simple as possible, it is assumed in this work that the quantity to be subtracted from the absolute length of a weir having complete contraction, to give its effective length, is directly proportional to the depth. It is also assumed that the quantity discharged by weirs of equal effective lengths, varies according to a constant power of the depth. There is no reason to think that either of these assumptions is perfectly correct; it will be seen, however, that they lead to results agreeing very closely with experiment

124. The formula proposed for weirs of considerable length in proportion to the depth upon them, and having complete contraction, is

$$Q = C(l - bnh)h^{a};$$ *

in which

$Q =$ the quantity discharged in cubic feet per second.
$C =$ a constant coefficient.
$l =$ the total length of the weir in feet.
$b =$ a constant coefficient.

---

* This formula was first suggested to the author by Mr. Boyden, in 1846.

n = the number of end contractions. In a single weir having complete contraction, n always equals 2, and when the length of the weir is equal to the width of the canal leading to it, $n = 0$.

h = the depth of water flowing over the weir, taken far enough upstream from the weir, to be unaffected by the curvature in the surface caused by the discharge.

a = a constant power.

The coefficient C can be determined only from experiments in which the actual discharge is known; the constants, a and b can, however, be determined without knowing the actual discharge in any particular case.

It has been stated that the proposed formula is applicable only to weirs having a considerable length in proportion to the depth of water running over them. It is found by experiment that, when the length equals or exceeds three times the depth, the formula applies; but in lengths less than this in proportion to the depth, the formula cannot be used with safety; the error increasing as the relative length of the weir diminishes.

It is evident, from the construction of the formula, that it cannot be of general application. The factor $l-bnh$ represents the *effective* length of the weir; if $l=bnh$ this effective length becomes 0, and the formula would give 0 for the discharge, which is absurd; similarly, if $bnh > l$, the discharge given by the formula would be negative. In weirs of very short length in proportion to the depth, the effect of the end contraction cannot be considered as independent of the length. The end contraction influences the discharge to a certain distance, A, from the end of a weir; if the whole length of the weir is greater than $2A$, the effect of the end contraction is independent of the length; but if the length is less than $2A$, the whole breadth of the stream is affected in its flow by the end contractions, and, consequently, the proposed formula would not apply.

In practical applications, this will seldom be an inconvenience, as it is nearly always practicable so to proportion the weir, that the length may not be less than three times the depth upon it; if, however, there is no end contraction, the proportion of the length to the depth is not material.

125. The author has made numerous experiments on the discharge of water over weirs, according to each of the methods described above.

First, those at the Tremont Turbine, and at the centre-vent water-wheel for moving the guard gates of the Northern Canal. In none of these experiments has any attempt been made to measure the absolute quantities flowing over the weirs; but simply to cause quantities of water known to be equal, to pass over weirs of different dimensions, noting the depth of water and length of weir in each case. From these data, as is explained above, certain factors in the formula can be determined.

Second, those at the Lower Locks, in which the absolute quantities passing over weirs of known dimensions, were measured directly.

As each of these three sets of experiments were made with different apparatus, they will be described separately.

EXPERIMENTS MADE AT THE TREMONT TURBINE, ON THE FLOW OF WATER OVER WEIRS.

126. The apparatus constructed to gauge the water discharged by the Tremont Turbine, with some modifications, was used for the experiments on the discharge over the weir; for a general description of this apparatus, see arts. 44, 45, and 46.

The experiments consisted in allowing a quantity of water, of unknown volume, to enter the wheelpit, through the turbine, the regulating gate of which was sufficiently opened for the purpose. This volume of water was then caused to flow over weirs of different dimensions, and the corresponding depth on the weir, assumed by the water in each experiment, was noted after the water had arrived at a uniform state.

The experiments are divided into series, in each of which the regulating gate was unchanged throughout, so that the apertures through which the water entered the wheelpit remained constant during each series.

Some variations necessarily occurred in the head acting upon these orifices; they were small, however, when compared to the whole head. The depths on the weir have been reduced, according to well-known principles, to what they would have been if the head had been constant. The leakage of the wheelpit also rendered another small correction necessary. After the corrections are made, we have in each series a collection of experiments in which the quantity discharged is the same, and we have also the requisite dimensions of the different weirs. These data, if perfectly accurate, are sufficient to enable us to determine, in the proposed formula for the discharge, the values of the constants a and b. It is not to be presumed, however, that the data are perfectly correct, but we can, at any rate, find the values of a and b that will give the most uniform results to the computed discharges in all the experiments in a series; the actual discharge being, by hypothesis, a constant quantity.

127. Some additions to the apparatus used in the experiments on the turbine were made for the weir experiments. The partitions, represented by figures

5, 6, and 7, plate V., were provided for the purpose of shortening or subdividing the weir. They were made of wood, faced on part of one side with plates of sheet-iron *a*, $\frac{3}{16}$ of an inch in thickness; the width *b c* was about 1.5 feet; the iron plate was two inches less One side of the timber *P*, figure 2, was in the same vertical plane as the upstream edge of the weir *H*. When the partitions were placed upon the weir, the top of them was supported by the timber *P*, and the bottom by the plate of iron *a*, which rested against the weir. Flashboards, represented by figures 8, 9, and 10, plate V., were also provided to close up portions of the weir; these, together with the partitions, were maintained in their respective positions, simply by the pressure of the water against them. Wherever leaks appeared at the joints of the partitions or flashboards, they were stopped with great ease and effect, by a little dough made of unbolted Indian meal, a handful of which was drawn over the upstream side of the joints; of course the orifices closed in this manner were very minute. In plate X., all the modifications of the weir produced by changing the partitions and flashboards, are represented; the several figures are referred to in column 8, table X. In the greater number of the experiments, two or more spaces were used at the same time; they were always of very nearly equal length, so that the length of each may be obtained by dividing the whole length of the weir given in column 6 by half the number of end contractions given in column 7.

The brackets *N*, figures 1 and 2, plate V., were placed on the downstream side of the weir, to support a board on which to stand for the purpose of adjusting the partitions and flashboards. The top of the board was about 9.5 inches below the top of the weir. In some of the experiments, a part of the sheet of water fell upon this board; in experiment 50 it was moved nearer to the weir, so that the entire sheet of water fell upon it, but without producing any sensible effect upon the discharge. In experiment 51, a three inch plank was placed on the top of the board, as is represented by the dotted lines at *O*, figure 2, plate V.; the effect of this obstruction, as indicated by the increased depth on the weir as measured by the hook gauge, was, to diminish the discharge, with the same depth on the weir, about $\frac{1}{1000}$.

It is to be regretted that the casting forming the sill of the weir, was not planed on its whole height on the side *H Q*, figure 4, plate V. When the weir was erected no thought was entertained of using it for these experiments, requiring, as they do, to be of value, to be free from all disturbing causes. The disturbance caused by the projection at *I*, can, however, have been scarcely sensible.

128. The data furnished by observation, together with the necessary reductions, and the results deduced from them, are contained in table X. Most of

the columns are sufficiently explained by the respective headings; several of them, however, require further explanation.

129. COLUMN 11. *Fall affecting the leakage of the wheelpit.* This is obtained by adding together the corresponding numbers in columns 9 and 10.

130. COLUMN 12. *Depth of water on the weir corrected for the leakage of the wheelpit.* This is obtained in the following manner.

It was clear, from the construction of the wheelpit, (art. 23,) that nearly the whole of the leakage passed through the wooden flooring, and that all the orifices through which it passed were constantly below the surface of the lower canal. In the construction of the wheelpit, no particular precautions were taken to prevent a free communication from the bottom of the wooden flooring to the lower canal; and as the amount of the leakage was very small, and the material, fine sand free from large springs, it is clear that the water could have had no appreciable obstruction after passing through the flooring, except from the pressure of the water in the lower canal. This being the case, the amount of the leakage would depend upon the head; or, in other words, upon the height from the surface of the water in the wheelpit, to the surface of the water in the lower canal. Let

L = the quantity of water leaking out of the wheelpit, in cubic feet per second.
A, A', A'', etc. = the areas of the several orifices through which the water passed.
C, C', C'', etc. = the corresponding coefficients of contraction.
h = the head, or the height from the surface of the water in the wheelpit, to the surface of the water in the lower canal. This head applies to all the orifices, as they are all below the surface of the water in the lower canal.

$$L = CA\sqrt{2gh} + C'A'\sqrt{2gh} + C''A''\sqrt{2gh} + \text{etc.};$$

$$L = (CA + C'A' + C''A'' + \text{etc.})\sqrt{2gh}.$$

The areas A, A', A'', etc., are constant, [illegible] are also the coefficients C, C', C'', etc., the variations in the head not being very great. Let

$$c = CA + C'A' + C''A'' + \text{etc.}:$$

then

$$L = c\sqrt{2gh} = c\sqrt{2g}\sqrt{h}.$$

The factor $c\sqrt{2g}$, being constant, can be determined by an experiment in which L and h are known. To determine this constant, the following experiment was made.

The weir was closed up by the flashboards, and made tight in the usual manner, so that no appreciable quantity passed over the weir; the head gate was closed, and the small quantity leaking through it was caught in the leak box and carried over the weir in the leak pipe (art. 24). The water in the wheelpit having then no supply, its surface began to lower, in consequence of the leakage through the floor; while thus falling, the following observations were made.

February 5, 1851, at 10h, 20′, 30″, A. M., the height of the water in the wheelpit above the top of the weir, was	0.596 feet.
And at 11h, 1′, 46″, A. M., the height was	0.396 "
Consequently the surface of the water in the wheelpit lowered in 2476″	0.200 feet.

The area of the surface of the water in the wheelpit, after making the proper deductions, was about 506 square feet; consequently,

$$L = \frac{506 \times 0.2}{2476} = 0.0409 \text{ cubic feet per second.}$$

During the interval of 2476 seconds, the mean height of the water in the lower canal was 1.2316 feet below the top of the weir, and the mean height in the wheelpit, during the same period, was 0.496 feet above the top of the weir, then

$$h = 1.2316 + 0.4960 = 1.7276 \text{ feet.}$$

Substituting these values of L and h in the equation

$$L = c\sqrt{2g}\sqrt{h},$$

we have

$$c\sqrt{2g} = 0.03112;$$

consequently,

$$L = 0.03112\sqrt{h}.$$

To find the depth on the weir, corrected for the leakage of the wheelpit, let

$h' =$ the depth on the weir by observation,

h'' = the depth on the weir corrected for the leakage,

l = length of the weir,

Q = the quantity passing over the weir, the dimensions being all in feet.

We have $Q+L$ = the total quantity entering the wheelpit, and which would have passed over the weir, if there had been no leakage out of the wheelpit.

To determine the corrected depth, it is necessary to assume some formula giving nearly the relations between the quantities h', l, and Q. Let us use that given by Lesbros[*] for a depth of 0.20 metres and complete contraction, which, when reduced to the English foot as the unit, and adopting our own notation, is

$$Q = 3.12\, l h'^{\frac{3}{2}};$$

we shall have also

$$Q + L = 3.12\, l h''^{\frac{3}{2}};$$

by subtraction

$$L = 3.12\, l h''^{\frac{3}{2}} - 3.12\, l h'^{\frac{3}{2}};$$

from which we derive

$$h'' = \left(h'^{\frac{3}{2}} + \frac{L}{3.12\, l}\right)^{\frac{2}{3}};$$

or substituting for L its value $0.03112\sqrt{h}$, we

$$h'' = \left(h'^{\frac{3}{2}} + \frac{0.03112\sqrt{h'}}{3.12\, l}\right)^{\frac{2}{3}}.$$

By this formula, the reduced heights given in column 12 have been obtained.

131. COLUMN 15. *Fall from the surface of water in the forebay, to the surface of the water in the wheelpit.* This is obtained by taking the difference of the corresponding numbers in columns 13 and 14.

132. COLUMN 16. *Uniform fall from the forebay to the wheelpit, to which the depths on the weir in each series are reduced.* The fall in the same series given in column 15, which is the nearest to the mean fall in all the experiments in the series, is assumed for this purpose; it is unimportant what fall is taken, provided it is near the mean.

133. COLUMN 17. *Depth on the weir corrected for the leakage of the wheelpit, and the variation in the fall.* It must be recollected that all the experiments of each

---

[*] *Expériences Hydrauliques sur les lois de l'écoulement de l'eau*, by M. Lesbros, Paris: 1851. Table XXXIX.

series, were made with the same opening of the regulating gate of the turbine; that is, the areas of the orifices through which the water entered the wheelpit, were the same in each. In all the experiments, a small quantity of the water entering the wheelpit, passed between the gate and the lower curb, in consequence of the leather packing not being perfectly adjusted; this did not affect the results, however, as these orifices were also submerged in the wheelpit. Under these circumstances, if the head had been constant, the quantity of water entering the wheelpit, would also have been constant; but the head was subject to a variation, comparatively small certainly, but sufficient to produce a material change in the quantity of water entering the wheelpit, and, consequently, in the depth on the weir.

To clear the results from this source of irregularity, it will be necessary to ascertain what the depths on the weir would have been, if the head had been constant. For this purpose, let

$H =$ the constant head to which the depths on the weir, in any particular series, are to be reduced, and which varies but little from the actual heads in the same series;
$H' =$ the actual head in the particular experiment to be reduced;
$h''' =$ the depth on the weir, corrected for the variation of the head, or corresponding to the constant head H;
$h'' =$ the depth on the weir corresponding to the head H', and which is the depth given by observation, corrected for the leakage of the wheelpit;
$q =$ the quantity of water, in cubic feet per second, that would have entered the wheelpit, if the head had been H;
$q' =$ the quantity of water corresponding to the head H', and which is the same as $Q + L$ (art 130);
$l =$ the length of the weir;
$C =$ the coefficient of the formula for the discharge over weirs;
a, a', a'', etc. $=$ the areas of the several orifices through which the water entered the wheelpit, all of them being submerged in the wheelpit;
c, c', c'', etc. $=$ the corresponding coefficients of contraction;

$$q' = ca\sqrt{2gH'} + c'a'\sqrt{2gH'} + c''a''\sqrt{2gH'} + \text{etc.};$$

$$q' = (ca + c'a' + c''a'' + \text{etc.})\sqrt{2gH'};$$

similarly

$$q=(ca+c'a'+c''a''+\text{etc.})\sqrt{2gH};$$

by division,

$$\frac{q'}{q}=\sqrt{\frac{H'}{H}};$$

also

$$q'=Clh''^{\frac{3}{2}} \text{ and } q=Clh'''^{\frac{3}{2}};$$

whence,

$$\frac{q'}{q}=\left(\frac{h''}{h'''}\right)^{\frac{3}{2}};$$

therefore,

$$\left(\frac{H'}{H}\right)^{\frac{1}{2}}=\left(\frac{h''}{h'''}\right)^{\frac{3}{2}} \text{ or } \left(\frac{H'}{H}\right)^{\frac{1}{3}}=\frac{h''}{h'''};$$

whence, we derive

$$h'''=h''\left(\frac{H}{H'}\right)^{\frac{1}{3}}.$$

By this last formula, the corrected depths given in column 17 have been computed.

By an inspection of column 18, it will be seen that the level of the water above the wheel was maintained throughout each series with great uniformity, excepting in a few experiments in which it was intentionally altered, as will be seen presently. The height of the water in the wheelpit necessarily varied with the depth upon the weir, and this is the principal cause of the variations in the fall.

Several of the experiments given in table X., were made for the express purpose of testing the accuracy of the method of reduction just described. Thus, in experiments 41 and 42, the weir was in the same state as in experiment 40, but the height of the water above the wheel was lowered, and the differences in the observed depths upon the weir, given in column 9, are to be attributed entirely to the diminution in the quantity of water entering the wheelpit, in consequence of the diminished head. If the method of reduction is accurate, however, the corrected depths in these three experiments, given in column 17, should be the same.

In table VIII, are collected all the experiments made for this object, together with the other experiments forming part of the corresponding series, with which they may be compared, the weir having been in the same state.

TABLE VIII.

Number of the experiment	Fall from the forebay to the wheelpit. Feet	Corrected depth upon the weir, in Feet	Variation in the fall from the original experiment. Feet	Variation in the corrected depth from the original experiment. Feet
40	14.088	0.79096		
41	13.554	0.79049	—0.531	—0.00047
42	13.143	0.78976	—0.989	—0.00120
49	13.304	0.95477		
52	13.136	0.95380	—0.468	—0.00097
53	12.362	0.95097	—0.912	—0.00380
60	13.719	1.13177		
61	12.806	1.12508	—0.913	—0.00669
72	13.816	0.92170		
73	13.315	0.92145	—0.501	—0.00025
74	12.665	0.92153	—1.151	—0.00017

It will be perceived that the variations in the fall, to which the method of reduction is applied in these experiments, are, nearly all of them, much greater than any that occur in the regular experiments. This was arranged for the purpose of applying an extreme test to the method. Several of the variations in the corrected depths, are not within the limits of ordinary observation; several of them, however, are sensible, and being all in the same direction, they cannot be attributed entirely to errors of observation, but, in part at least, to either a slight defect in the method of reduction, or to the instability of the apparatus.

It was observed during the course of the experiments, that the quantity of water entering the wheelpit, sometimes diminished sensibly, although no change had been made in the height of the regulating gate; the precaution having been taken to fix, in a secure manner, the apparatus by which the gate was moved. At the time the experiments were made, this change was attributed to a minute lowering of the gate, taking place very slowly, and arising from a defect in the stiffness of the apparatus, aided by a slight, but not totally insensible vibration of the whole apparatus, caused by the passage of the water through the apertures. To show how minute a change in the height of the regulating gate, would produce the observed changes in the quantity, let us take the two first experiments given in table IX The regulating gate was raised to a height not exceeding 0 01 feet; supposing it to have been at just that height, and that any change in its height would have produced an equal proportional change in the discharge, the observed proportional change in the quantity was 0.00046; consequently, the absolute change in the height of the gate must have been 0.0000046 feet.

In order to prevent this source of irregularity from affecting the experiments, the regulating gate was usually set some hours before the experiments were made. This probably obviated the difficulty in part, but not entirely, as will be seen by table IX., in which are collected all the experiments that were repeated under identical circumstances.

TABLE IX.

Number of the experiment	Number of the series	Corrected depth upon the weir in feet	Variation in the depth from the first experiment, in feet	Proportional change in the quantity [illegible] entered the weir [illegible]	Time that had elapsed when the experiment was made after the gate was set	
					Hours	Minutes
3	I.	0.19583				
7	"	0.19577	—0.00006	—0.00040		
8	II.	0 28386	. . . .	. . . .	0	22
12	"	0.28505	+0.00119	+0.00761	1	31
16	III.	0.29223	. . . .	. . . .	4	34
20	"	0.29166	—0 00057	—0 00292	5	39
24	"	0.29210	—0.00013	—0 00067	6	30
26	IV.	1.06532	. . . .	. . . .	16	58
30	"	1.06548	+0.00016	+0.00023	17	51
35	V.	0 79100	. . . .	. . . .	2	25
40	"	0.79006	—0.00094	—0.00178	3	3
44	VI.	0.95656	. . . .	. . . .	5	20
49	"	0 95477	—0.00179	—0.00281	6	40
55	VII.	1.18356	. . . .	. . . .	2	20
58	"	1.18306	—0.00050	—0.00063	3	31
63	"	1.18177	—0 00179	—0.00227	4	30
66	VIII.	1 06358	. . . .	. . . .	3	08
69	"	1.06272	—0 00086	—0.00121	3	46
Mean proportional change in the quantity, neglecting the signs, ...				0.00208		

Although the variations in the depths given in the preceding table are very small, the fact that they are nearly all negative precludes the idea that they are entirely due to errors of observation; we must, therefore, attribute to some other cause a portion of the irregularity.

134. COLUMN 19. *Combination of experiments used to determine the value of a.* It has been shown (art. 122) how, by means of two experiments in which the quantities passing over different weirs are equal, we may determine a in the formula

$$Q = Clh^a.$$

We now propose to show how, by means of two such experiments, the value of a may be found in the proposed formula

$$Q = C(l - bnh)h^a.$$

In this equation, we have b and a constant quantities to be determined; we have also C a constant, which we may here consider as indeterminate; the same may be said of Q, as limited to the experiments in the same series.

Let l, n, and h, represent the length of the weir, the number of end contractions, and the depth upon the weir in one experiment; and l_1, n_1, and h_1, the corresponding quantities in another experiment of the same series; we have

$$Q = C(l - bnh)h^a;$$

and

$$Q = C(l_1 - bn_1h_1)h_1^a:$$

since for the same series Q is constant, we have

$$(l - bnh)h^a = (l_1 - bn_1h_1)h_1^a:$$

taking the logarithms,

$$a \text{ Log. } h + \text{Log. } (l - bnh) = a \text{ Log. } h_1 + \text{Log. } (l_1 - bn_1h_1):$$

whence we derive

$$a = \frac{\text{Log. } (l_1 - bn_1h_1) - \text{Log. } (l - bnh)}{\text{Log. } h - \text{Log. } h_1}.$$

This equation is still indeterminate, but can be rendered determinate, by assuming a value for b.

If the formula represents the true law, and the experiments from which the values of the constants are to be derived are perfectly accurate, the particular combination of experiments to be used is evidently unimportant. As such an assumption would be very unreasonable, we have combined the experiments, with a view of obtaining the best approximation from imperfect data; and this we have accomplished by selecting experiments the most remote from each other in the values of the respective data they furnish; thus, in series I., the combinations are made by combining experiment 6, in which l has the least, and, consequently, h the greatest value, with each of the others, omitting entirely all the experiments which, for any reason, appear to be unsuitable.

Generally, in each series, one experiment has been repeated [illegible] a test, in order to show if any change had taken place in the apparatus; thus, in series III, experiments 16, 20, and 24, were made, so far as is known, under identical circumstances; in such cases, means deduced from the repeated experiments have been used instead of making a separate combination with each.

135. Columns numbered 20 to 25. Values of a when $b = 0.07$, $b = 0.065$, etc The object is, to find the values of a and b, in the formula

$$Q = C(l - bnh)h^a,$$

that will give to the computed discharges in each series the most uniform results. For this purpose, successive values of b are assumed, and the corresponding values of a, determined. The value of b leading to values of a, having the least variation among themselves, will evidently be that most nearly fulfilling this condition. To aid in the selection of the proper value of b, the table gives the differences between the values of a deduced from each combination, and the mean value of a deduced from all the combinations, with the same value of b, and the sums of these differences (having no regard to the sign) are also given. It will be seen that the sum of the differences is least when the value of $b = 0.05$, the corresponding mean value of a being 1.46994, or 1.47 very nearly; consequently, to represent the whole of the experiments with the most uniformity, the formula becomes

$$Q = C(l - 0.05nh)h^{1.47}.$$

TABLE X.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE.

1	2	3		4				5	6	7	8	9	10	11
Number of the series and of the experiment.	Date of the experiment. 1851	Temperature of the atmosphere in degrees of Fahrenheit's thermometer		TIME				Duration of the experiment, in minutes.	Total length of the weir in feet. l	No. of the end contractions. n.	Reference to the figures in Plate X.	Depth of water on the weir by experiment in feet. H	Height of the water in the weir canal below the top of the weir in feet	Fall [illegible] in feet. h
		External, in the shade	Near the weir	Beginning of the experiment		Ending of the experiment								
				H.	min	H.	min							
Series I.		*												
Exp. 1	January 30, A.M.			10	0	10	12	12	6.987	2	Fig. 1	0.4125	1.10	1.47
" 2	" " "			10	18	10	25	7	13.978	4	" 2	0.4518	1.10	1.35
" 3	" " "	6.50		10	39	10	46.5	7.5	13.978	8	" 3	0.4952	1.10	1.30
" 4	" " "	[illegible]	31.50	11	2	11	6	4	10.482	6	" 4	0.2889	1.10	1.10
" 5	" " "	5.75	31.00	11	20	11	26	6	7.000	4	" 5	[illegible]	1.17	1.18
" 6	" " "	6.25		11	40	11	45	5	8.500	2	" [illegible]	0.6028	1.25	1.75
" 7	" " "	5.75		11	52	11	55	3	13.978	8	" 3	0.4051	1.22	1.42
Series II.														
Exp. 8	January 30, P.M.		30.75	2	22	2	26	4	13.978	8	Fig. 3	0.2310	1.10	1.33
" 9	" " "	4.50	30.50	2	35.5	2	41	5.5	10.482	6	" 4	0.2812	1.10	1.48
" 10	" " "	4.50	30.50	2	54	3	0	6	7.000	4	" 5	0.4788	1.10	1.17
" 11	" " "	4.25		3	15	3	21	6	8.500	2	" 6	0.5473	1.20	1.80
" 12	" " "	4.00	30.75	3	31	3	38	7	13.978	8	" [illegible]	0.2411	1.27	1.50
" 13	" " "	3.50	30.50	3	53	3	59	6	13.978	4	" 2	0.2340	1.22	1.15
" 14	" " "			4	11	4	16.5	5.5	6.987	2	" 1	0.3710	1.10	1.17
" 15	" " "	2.75	30.75	4	24	4	32	8	16.980	4	" 7	0.2016	1.00	1.29
Series III.														
Exp. 16	January 31, P.M.	5.00	31.00	2	23.5	2	32	8.5	13.978	4	Fig. [illegible]	0.2916	1.17	[illegible]
" 17	" " "	5.00	31.25	2	41	2	49.5	8.5	6.987	2	" 1	[illegible]	1.17	[illegible]
" 18	" " "	5.00	31.25	2	56.5	3	5	8.5	13.978	8	" 3	[illegible]	[illegible]	1.15
" 19	" " "	5.00	31.00	3	12	3	18	6	10.481	6	" 4	[illegible]	1.13	1.13
" 20	" " "	5.00	30.75	3	29.5	3	35.5	6	13.978	4	" 2	0.2910	1.12	1.11
" 21	" " "	4.50	30.50	3	46	3	53	7	[illegible]	4	" 5	0.4681	1.15	1.62
" 22	" " "	4.50		4	2.5	4	8.5	6	8.500	2	" 8	0.7178	1.12	1.87
" 23	" " "	4.25	31.00	4	14	4	20	6	16.980	4	" 7	0.2548	1.12	1.37
" 24	" " "	4.00		4	29.5	4	35.5	6	13.978	4	" 2	0.2914	1.16	1.15
Series IV.														
Exp. 25	February 1, A.M.	5.00	31.00	9	15	9	21	6	13.978	4	Fig. 2	0.4071	1.11	1.55
" 26	" " "	7.00		9	33.5	9	41	7.5	8.496	2	" 8	1.0117	1.12	2.16
" 27	" " "	[illegible]	30.00	9	52	9	57	5	[illegible]	4	" 5	0.6577	1.15	1.81
" 28	" " "	10.00		10	4	10	11	7	10.481	6	" 4	0.4177	1.10	1.60
" 29	" " "	10.00	30.50	10	16	10	21	5	13.978	8	" 3	0.4030	1.15	1.56
" 30	" " "	10.50		10	31.5	10	39.5	8	8.496	2	" 8	[illegible]	1.17	2.22
" 31	" " "	14.25	31.50	10	57	11	1	4	8.496	2	" 8	1.0152	1.12	2.17
" 32	" " "	15.00		11	19	11	24.5	5.5	6.987	2	" 1	[illegible]	1.17	1.82
" 33	" " "	15.50	30.75	11	29.5	11	36	6.5	16.980	4	" 7	0.3576	1.22	1.58

TABLE X—Continued.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE

Number of the series and of the experiment.	12 Depth of water on the weir, corrected for the leakage of the wheelpit, in feet. *h'*.	13 Height of water above the weir, taken in the forebay, in feet.	14 Height of the water in the wheelpit, in feet.	15 Fall from the surface of the water in the forebay to the surface of the water in the wheelpit, in feet. *H'*.	16 Uniform fall from the forebay to the wheelpit, to which the depths upon the weir in each series are reduced, in feet. *H*.	17 Depth on the weir, corrected for the leakage of the wheelpit and the variation in the fall, in feet. *h''*.	18 REMARKS.
Series I.							
Exp. 1	0.31456	14.869	0.320	14.549	14.549	0.31456	In experiments 2, 3, and 7, the contraction was incomplete, as the water followed the top of the weir.
" 2	0.19605	14.896	0.211	14.685	"	0.19540	
" 3	0.19645	14.894	0.205	14.689	"	0.19588	
" 4	0.24048	14.881	0.217	14.664	"	0.23997	
" 5	0.31696	14.892	0.320	14.572	"	0.31679	
" 6	0.50634	14.876	0.510	14.366	"	0.50848	
" 7	0.19688	14.886	0.200	14.686	"	0.19577	
Series II.							
Exp. 8	0.23414	14.910	0.240	14.670	14.619	0.23386	In experiment 15 the contraction was incomplete, as the water followed the top of the weir.
" 9	0.28560	14.909	0.290	14.619	"	0.28560	
" 10	0.37568	14.915	0.380	14.535	"	0.37640	
" 11	0.60059	14.916	0.610	14.306	"	0.60134	
" 12	0.23630	14.906	0.240	14.666	"	0.23505	
" 13	0.23419	14.912	0.240	14.672	"	0.23390	
" 14	0.37370	14.910	0.380	14.530	"	0.37455	
" 15	0.20558	14.918	0.210	14.708	"	0.20517	
Series III.							
Exp. 16	0.29236	14.897	0.300	14.597	14.582	0.29223	
" 17	0.46608	14.900	0.470	14.430	"	0.46808	
" 18	0.29126	14.867	0.300	14.567	"	0.29382	
" 19	0.35770	14.897	0.365	14.582	"	0.35770	
" 20	0.29205	14.890	0.300	14.590	"	0.29166	
" 21	0.47017	14.887	0.477	14.410	"	0.47149	
" 22	0.75080	14.888	0.760	14.128	"	0.75798	
" 23	0.25571	14.878	0.260	14.618	"	0.25520	
" 24	0.29210	14.886	0.300	14.586	"	0.29210	
Series IV.							
Exp. 25	0.40803	14.904	0.420	14.484	14.537	0.40852	In experiments 26 and 30, the water flowing over the weir fell upon a board placed upon the brackets N, figures 1 and 2, plate V.; in experiment 31 the board was removed. So far as is known the three experiments were identical in all other respects. By comparing the corrected depths upon the weir given in column 17, it appears that the board offered no appreciable obstruction to the discharge.
" 26	1.04743	14.877	1.060	13.817	"	1.06532	
" 27	0.65928	14.871	0.670	14.201	"	0.66444	
" 28	0.49884	14.877	0.510	14.367	"	0.50080	
" 29	0.41053	14.893	0.420	14.473	"	0.41113	
" 30	1.04837	14.908	1.060	13.848	"	1.06548	
" 31	1.04734	14.886	1.060	13.826	"	1.06560	
" 32	0.65099	14.904	0.660	14.244	"	0.65543	
" 33	0.35842	14.907	0.370	14.537	"	0.35842	

TABLE X. Continued.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE.

Number of the series and of the experiment	19	20			21			22		
		b = 0.07			b = 0.065			b = 0.06		
	Combinations of experiments used to determine the values of a	Values of a	Differences from the mean value of a or from 1.47797		Values of a	Differences from the mean value of a or from 1.47595		Values of a	Differences from the mean value of a or from 1.47394	
			+	−		+	−		+	−
Series I.										
Exp 1	1 and 6	1.4691		0.00887	1.4669		0.00905	1.4648		0.00914
" 2										
" 3										
" 4	4 " 6	1.4753		0.00267	1.4742		0.00175	1.4731		0.00084
" 5	5 " 6	1.4814	0.00343		1.4801	0.00415		1.4780	0.00406	
" 6										
" 7										
Series II.										
Exp 8	8, 12, and 11	1.4768		0.00117	1.4756		0.00035	1.4744	0.00046	
" 9	9 " 11	1.4787	0.00073		1.4775	0.00155		1.4763	0.00236	
" 10	10 " 11	1.4805	0.00253		1.4791	0.00315		1.4777	0.00376	
" 11										
" 12	8, 12, " 11	1.4768		0.00117	1.4756		0.00035	1.4744	0.00046	
" 13	13 " 11	1.4781	0.00013		1.4766	0.00065		1.4751	0.00116	
" 14	14 " 11	1.4774		0.00057	1.4748		0.00115	1.4723		0.00164
" 15	15 " 11	1.4800	0.00203		1.4786	0.00265		1.4772	0.00326	
Series III.										
Exp 16	16, 20, 24, and 22	1.4778		0.00017	1.4759		0.00005	1.4740	0.00006	
" 17	17 " 22	1.4784	0.00043		1.4752		0.00075	1.4721		0.00184
" 18	18 " 22	1.4811	0.00313		1.4797	0.00375		1.4782	0.00426	
" 19	19 " 22	1.4827	0.00473		1.4811	0.00515		1.4795	0.00556	
" 20	16, 20, 24, " 22	1.4778		0.00017	1.4759		0.00005	1.4740	0.00006	
" 21	21 " 22	1.4814	0.00343		1.4796	0.00365		1.4778	0.00386	
" 22										
" 23	23 " 22	1.4752		0.00277	1.4734		0.00255	1.4716		0.00234
" 24	16, 20, 24, " 22	1.4778		0.00017	1.4759		0.00005	1.4740	0.00006	
Series IV.										
Exp 25	25 and 26, 30	1.4827	0.00473		1.4800	0.00405		1.4773	0.00336	
" 26										
" 27	27 " 26, 30	1.5023	0.02433		1.4997	0.02375		1.4971	0.02316	
" 28	28 " 26, 30	1.4857	0.00773		1.4831	0.00715		1.4812	0.00726	
" 29	29 " 26, 30	1.4838	0.00583		1.4817	0.00575		1.4796	0.00566	
" 30										
" 31										
" 32	32 " 26, 30	1.4878	0.00983		1.4832	0.00725		1.4786	0.00466	
" 33	33 " 26, 30	1.4853	0.00733		1.4828	0.00685		1.4802	0.00626	

TABLE X—CONTINUED

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE

Number of the series and of the experiment	23 b = 0.055 Values of a	Differences from the mean value of a, or from 1.47194 +	−	24 b = 0.05 Values of a	Differences from the mean value of a, or from 1.46994 +	−	25 b = 0.045 Values of a	Differences from the mean value of a, or from 1.46795 +	−
Series I. Exp. 1	1.4626		0.00934	1.4605		0.00944	1.4584		0.00955
" 2									
" 3									
" 4	1.4721	0.00016		1.4710	0.00106		1.4700	0.00205	
" 5	1.4778	0.00586		1.4765	0.00656		1.4754	0.00745	
" 6									
" 7									
Series II. Exp. 8	1.4738	0.00186		1.4722	0.00226		1.4710	0.00305	
" 9	1.4750	0.00306		1.4737	0.00376		1.4725	0.00455	
" 10	1.4762	0.00426		1.4749	0.00496		1.4734	0.00545	
" 11									
" 12	1.4738	0.00186		1.4722	0.00226		1.4710	0.00305	
" 13	1.4735	0.00156		1.4721	0.00216		1.4706	0.00265	
" 14	1.4697		0.00224	1.4671		0.00284	1.4646		0.00335
" 15	1.4758	0.00386		1.4744	0.00446		1.4730	0.00505	
Series III. Exp. 16	1.4721	0.00016		1.4702	0.00026		1.4688	0.00085	
" 17	1.4688		0.00314	1.4656		0.00434	1.4624		0.00555
" 18	1.4748	0.00286		1.4753	0.00536		1.4739	0.00595	
" 19	1.4780	0.00606		1.4764	0.00646		1.4748	0.00685	
" 20	1.4721	0.00016		1.4702	0.00026		1.4683	0.00035	
" 21	1.4760	0.00406		1.4742	0.00426		1.4725	0.00455	
" 22									
" 23	1.4699		0.00204	1.4681		0.00184	1.4663		0.00165
" 24	1.4721	0.00016		1.4702	0.00026		1.4688	0.00085	
Series IV. Exp. 25	1.4746	0.00266		1.4720	0.00206		1.4693	0.00135	
" 26									
" 27	1.4945	0.02256		1.4920	0.02206		1.4895	0.02155	
" 28	1.4789	0.00696		1.4767	0.00676		1.4745	0.00655	
" 29	1.4776	0.00566		1.4755	0.00556		1.4735	0.00555	
" 30									
" 31									
" 32	1.4741	0.00216		1.4695		0.00044	1.4651		0.00285
" 33	1.4777	0.00576		1.4752	0.00526		1.4728	0.00485	

TABLE X.—CONTINUED.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE.

1	2	3		4				5	6	7	8	9	10	1
Number of the series and of the experiment.	Date of the experiment, 1851.	Temperature of [illegible] Fahrenheit's thermometer.		TIME.				[illegible] of the experiment [illegible]	Total length of the weir, in feet. l	No. of [illegible] n	Reference to figures in plate X.	Depth of water on the weir [illegible] in feet H	Height of [illegible] weir, in feet	[illegible]
				Beginning of the experiment.		Ending of the experiment.								
		In air in shade.	Near the [illegible]	h	m	h	m							
Series V.														
Exp. 34	February 1, P.M.	20.75	31.50	2	11	2	15.5	4.5	13.978	1	Fig. 2	0.1937	1.12	1.01
" 35	" " "	20.00	31.50	2	25	2	33	8	6.987	2	" 1	0.7168	1.16	1.95
" 36	" " "	21.50	31.50	2	31	2	43	4	13.978	8	" 3	0.1981	1.17	1.67
" 37	" " "	22.25	31.50	2	39.5	2	51	15	10.981	6	" 4	0.1[illegible]	1.2.	.81
" 38	" " "	20.50		3	3.5	3	11	10.5	6.989	1	" 5	0.8000	1.21	2.01
" 39	" " "	21.50	30.00	3	19	3	21.5	4.5	.980	4	" 7	0.1147	1.22	1.65
" 40	" " "	21.00	31.50	3	31	3	18	11	6.987	2	" 1	0.7806	1.21	2.03
" 41	" " "	18.50	31.25	4	16	4	26	10	6.987	2	" 1	0.7790	1.25	2.04
" 42	" " "	18.00		4	52	5	0	8	6.987	2	" 1	0.7701	1.25	2.12
Series VI.														
Exp. 43	February 3, P.M.	38.25		2	0	2	11	5	13.978	4	Fig. 2	0.5977	1.10	1.70
" 44	" " "	38.25	32.25	2	20.5	2	33	9.5	6.987	2	" 1	0.4561	1.17	2.14
" 45	" " "	38.25		2	37	2	43	6	6.987	4	" 9	0.9636	1.17	2.13
" 46	" " "	37.50	32.00	2	48.5	2	56	7.5	13.978	8	" 3	0.6023	1.10	1.74
" 47	" " "	37.50		3	6.5	3	13	6.5	10.188	6	" 4	0.7308	1.11	1.84
" 48	" " "	37.25	32.00	3	16.5	3	22.5	6	10.980	4	" 7	0.5218	1.13	1.65
" 49	" " "	37.00		3	40	3	45	5	6.987	2	" 1	0.9544	1.14	2.08
" 50	" " "			3	47	3	5.	12	6.987	2	" 1	0.9531	1.14	2.08
" 51	" " "			4	2	4	4	2	6.987	2	" 1	0.9539	1.13	2.08
" 52	" " "			4	21	4	31	8	6.987	2	" 1	0.9115	1.18	2.07
" 53	" " "	38.75		4	46.5	5	0	13.5	6.987	2	" 1	0.9275	1.14	2.07
Series VII.														
Exp. 54	February 4, A.M.	26.00	31.75	9	0	9	12.5	6.5	10.980	4	Fig. 7	0.5233	1.12	1.61
" 55	" " "	26.50		9	26.5	9	37	10.5	5.187	2	" 10	1.1278	1.11	2.27
" 56	" " "	31.00	31.75	9	41	9	57	16	6.987	2	" 11	0.9511	1.15	2.10
" 57	" " "	30.00	31.75	10	17	10	22	5	8.489	2	" 12	0.8375	1.14	8
" 58	" " "	31.25	31.75	10	31	10	36	5	5.187	2	" 10	1.1260	1.17	2.30
" 59	" " "	33.50	31.75	10	42	10	46.5	4.5	6.987	4	" 13	0.[illegible]09	1.14	2.1[illegible]
" 60	" " "	34.00	31.75	10	56.5	11	1	4.5	13.978	8	" 3	0.6017	1.14	1.73
" 61	" " "	35.00		11	10	11	15	5	10.180	6	" 4	0.7103	1.12	1.85
" 62	" " "	37.00	31.75	11	20	11	26	6	13.978	4	" 2	0.5371	1.15	1.75
" 63	" " "	38.00	31.75	11	39.5	11	55	15.5	5.187	[illegible]	" 10	1.1256	1.1[illegible]	2.27
" 64	" " P.M.	40.75		0	16	0	25	9	5.187	2	" 10	1.0935	1.13	2.22
Series VIII.														
Exp. 65	February 4, P.M.	38.25	32.00	3	10.5	3	14	3.5	10.980	4	Fig. 7	0.2910	1.19	1.12
" 66	" " "	37.50		3	33.5	3	40	6.5	7.829	2	" 11	1.0581	1.17	2.28
" 67	" " "	37.00		3	45	3	52	7	8.658	4	" 15	0.8650	1.18	1.81
" 68	" " "	36.75		3	58	4	2	4	5.487	6	" 16	0.5010	1.21	1.75
" 69	" " "			4	11.5	4	16	4.5	7.829	2	" 14	1.0574	1.21	2.27
" 70	" " "	36.25	32.00	4	21	4	25	4	8.489	2	" 12	0.[illegible]706	1.19	1.56
" 71	" " "			4	32	4	37	5	5.487	[illegible]	" 10	0.4980	1.19	1.6[illegible]
Series IX.														
Exp. 72	February 4, P.M.	31.25		4	53	5	2	9	10.980	4	Fig. 7	0.9206	1.18	2.10
" 73	" " "	34.25		5	17.5	5	24	6.5	10.980	4	" 7	0.9091	1.02	1.98
" 74	" " "	33.75		5	31	5	43	12	10.980	4	" 7	0.8911	1.17	2.06

TABLE X—CONTINUED.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE

Number of the series and of the experiment	12 Depth of water on the weir, corrected for the leakage of the wheel pit, in feet. h'.	13 Height of water above the weir taken in the forebay, in feet	14 Height of water in the wheel pit, in feet	15 Fall from the surface of water in the forebay to the surface of the water in the wheel pit, in feet. H'.	16 Mean fall from the forebay to the wheel, to which the depths on the weir in each series are reduced, in feet. H	17 Depth on the weir, corrected for the leakage of the wheel pit, and the variation in the fall, in feet. h''	18 REMARKS
Series V.							
Exp. 34	0.19156	14.815	0.510	14.305	14.079	0.19160	In experiments 41 and 42 the weir was in the same state as in experiment 40; the height of the water above the wheel was reduced for the purpose of testing the method of reduction.
" 35	0.79229	14.910	0.810	14.100	"	0.79190	
" 36	0.49897	14.891	0.520	14.371	"	0.49557	
" 37	0.60710	14.891	0.620	14.271	"	0.60137	
" 38	0.80151	14.899	0.820	14.079	"	0.80151	
" 39	0.48116	14.908	0.450	14.458	"	0.48068	
" 40	0.79112	14.847	0.800	14.047	"	0.79096	
" 41	0.78051	14.352	0.798	13.554	"	0.79043	
" 42	0.77198	13.939	0.790	13.149	"	0.78976	
Series VI.							
Exp. 43	0.59860	14.903	0.620	14.283	13.907	0.59920	In experiments 50 and 51 the weir was in the same state as in experiment 49, excepting that in 50 a board was placed on the brackets *N*, figs 1 and 2, plate V, on which the water fell; and in exp. 51, the plank *O*, fig. 2, plate V, was placed in the position represented; the top of the plank was [illegible] 5 inches below the top of the weir. In exps. 52 and 53, the weir was in the same state as in exp. 49, the height of the water above the wheel was lowered for the purpose of testing the method of reduction.
" 44	0.95752	14.929	0.980	13.949	"	0.95656	
" 45	0.96501	14.915	0.992	13.923	"	0.96401	
" 46	0.60311	14.891	0.630	14.261	"	0.59804	
" 47	0.73181	14.887	0.755	14.132	"	0.72790	
" 48	0.52449	14.889	0.550	14.339	"	0.51917	
" 49	0.95471	14.881	0.980	13.901	"	0.95477	
" 50	0.95161	14.886	0.980	13.906	"	0.95159	
" 51	0.95580	14.887	0.980	13.907	"	0.95580	
" 52	0.91291	14.406	0.970	13.436	"	0.95980	
" 53	0.92832	13.914	0.952	12.962	"	0.95097	
Series VII.							
Exp. 54	0.52399	14.889	0.550	14.339	13.882	0.51837	In experiments 63 and 64 the weir was in the same state, as in experiment 62; the height of the water above the wheel was lowered for the purpose of testing the method of reduction.
" 55	1.12952	14.881	1.150	13.731	"	1.13356	
" 56	0.95581	14.882	0.980	13.902	"	0.95595	
" 57	0.83870	14.875	0.865	14.010	"	0.836[illegible]4	
" 58	1.12813	14.872	1.152	13.720	"	1.13306	
" 59	0.96230	14.872	0.990	13.882	"	0.96280	
" 60	0.60251	14.868	0.629	14.239	"	0.59743	
" 61	0.73131	14.865	0.754	14.111	"	0.72739	
" 62	0.59791	14.860	0.620	14.240	"	0.59286	
" 63	1.12732	14.869	1.150	13.719	"	1.13177	
" 64	1.08523	13.926	1.120	12.806	"	1.12508	
Series VIII.							
Exp. 65	0.23257	14.894	0.210	14.684	13.839	0.22817	In experiments 66, 67, 68, and 69, the heights of the several bays of the weir were deemed to be too short relative to the depth flowing over, for the proposed formula to apply.
" 66	1.06317	14.889	1.068	13.821	"	1.06358	
" 67	0.66802	14.895	0.676	14.219	"	0.66202	
" 68	0.50885	14.902	0.515	14.387	"	0.50241	
" 69	1.06272	14.905	1.066	13.839	"	1.06272	
" 70	0.37221	14.905	0.380	14.525	"	0.36625	
" 71	0.50023	14.909	0.595	14.314	"	0.49860	
Series IX.							
Exp. 72	0.92119	14.864	1.048	13.816	13.839	0.92170	Experiments 72, 73, and 74 were made for the express purpose of testing the mode of reduction.
" 73	0.90967	14.950	1.035	13.915	"	0.92145	
" 74	0.89169	13.678	1.013	12.665	"	0.92153	

TABLE X—CONTINUED.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE

Number of the series and of the experiment.	19	20			21			22		
		b = 0.07			b = 0.08			b = 0.09		
	Combinations of experiments used to determine the value of *a*.	Values of *a*.	Differences from the mean value of *a*, 1.47797.		Values of *a*.	Differences from the mean value of *a*, 1.47595.		Values of *a*.	Differences from the mean value of *a*, 1.47401.	
			+	−		+	−		+	−
Series V.										
Exp. 31	31 and 38	1.4665		0.01147	1.4641		0.01185	1.4577		0.01631
" 35	35, 40, " 39	1.4737		0.00427	1.4726		0.00335	1.4711		0.00291
" 36	36 " 38	1.4679		0.01007	1.4660		0.00995	1.4641		0.00991
" 37	37 " 38	1.4651		0.01287	1.4630		0.01295	1.4610		0.01291
" 38										
" 39	39 " 38	1.4700		0.00797	1.4670		0.00895	1.4640		0.00991
" 40	35, 40, " 39	1.4737		0.00427	1.4726		0.00335	1.4711		0.00291
" 41										
" 42										
Series VI.										
Exp. 43	43 and 45	1.4827	0.00473		1.4785	0.00255		1.4741	0.00009	
" 44	44, 49, " 48	1.4706		0.00737	1.4694		0.00655	1.4681		0.00591
" 45										
" 46	46 " 45	1.4821	0.00413		1.4798	0.00385		1.4771	0.00309	
" 47	47 " 46	1.4890	0.01103		1.4870	0.01105		1.4850	0.01099	
" 48	48 " 47	1.4879	0.00993		1.4834	0.00745		1.4788	0.00479	
" 49	44, 49, " 48	1.4706		0.00737	1.4694		0.00655	1.4681		0.00591
" 50										
" 51										
" 52										
" 53										
Series VII.										
Exp. 54	54 and 55, 58, 63	1.4715		0.00647	1.4696		0.00635	1.4677		0.00631
" 55										
" 56	56 and 71	1.4699		0.00807	1.4686		0.00735	1.4671		0.00691
" 57	57 and 55, 58, 63	1.4881	0.01013		1.4843	0.00835		1.4806	0.00659	
" 58										
" 59	59 and 71	1.4850	0.00703		1.4811	0.00515		1.4778	0.00379	
" 60	60 and 55, 58, 63	1.4695		0.00847	1.4689		0.00705	1.4681		0.00591
" 61	61 " 55, 58, 63	1.4619		0.01607	1.4620		0.01395	1.4620		0.01201
" 62	62 " 55, 58, 63	1.4711		0.00687	1.4691		0.00685	1.4671		0.00691
" 63										
" 64										
Series VIII.										
Exp. 65	65 and 71	1.4755		0.00247	1.4747		0.00125	1.4740		0.00001
" 66										
" 67										
" 68										
" 69										
" 70	70 " 71	1.4845	0.00653		1.4829	0.00695		1.4813	0.00729	
" 71										
Series IX.										
Exp. 72										
" 73										
" 74										
Sums of the differences and mean values of *a*.		1.47797	0.26767		1.47595	0.25085		1.47401	0.23672	

TABLE X—CONTINUED.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE TREMONT TURBINE.

Number of the series and of the experiment.	23			24			25		
	$b = 0.055$			$b = 0.05$			$b = 0.045$		
	Values of a	Differences from the mean value of a, or from 1.47194		Values of a	Differences from the mean value of a, or from 1.46994		Values of a	Differences from the mean value of a, or from 1.46795	
		+	−		+	−		+	−
Series V.									
Exp. 34	1.4548		0.01764	1.4510		0.01894	1.4473		0.02085
" 35	1.4706		0.00134	1.4695		0.00044	1.4684	0.00045	
" 36	1.4622		0.00974	1.4602		0.00974	1.4584		0.00955
" 37	1.4589		0.01304	1.4567		0.01324	1.4547		0.01325
" 38									
" 39	1.4610		0.01094	1.4581		0.01184	1.4551		0.01285
" 40	1.4706		0.00134	1.4695		0.00044	1.4684	0.00045	
" 41									
" 42									
Series VI.									
Exp. 43	1.4703		0.00164	1.4602		0.00974	1.4622		0.00575
" 44	1.4668		0.00514	1.4656		0.00434	1.4643		0.00365
" 45									
" 46	1.4751	0.00316		1.4728	0.00286		1.4705	0.00255	
" 47	1.4830	0.01106		1.4811	0.01116		1.4790	0.01105	
" 48	1.4741	0.00216		1.4699		0.00004	1.4654		0.00255
" 49	1.4668		0.00514	1.4656		0.00434	1.4618		0.00365
" 50									
" 51									
" 52									
" 53									
Series VII.									
Exp. 54	1.4657		0.00624	1.4698		0.00014	1.4619		0.00605
" 55									
" 56	1.4661		0.00584	1.4649		0.00504	1.4636		0.00435
" 57	1.4760	0.00406		1.4733	0.00336		1.4696	0.00165	
" 58									
" 59	1.4742	0.00226		1.4706	0.00066		1.4670		0.00095
" 60	1.4677		0.00424	1.4672		0.00274	1.4666		0.00135
" 61	1.4620		0.00994	1.4621		0.00784	1.4621		0.00585
" 62	1.4652		0.00674	1.4633		0.00664	1.4613		0.00665
" 63									
" 64									
Series VIII.									
Exp. 65	1.4731	0.00116		1.4722	0.00226		1.4714	0.00345	
" 66									
" 67									
" 68									
" 69									
" 70	1.4797	0.00776		1.4782	0.00826		1.4765	0.00855	
" 71									
Series IX.									
Exp. 72									
" 73									
" 74									
		0.23124			0.22000			0.23945	
	1.47194			1.46994			1.46795		

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE CENTRE-VENT WHEEL FOR MOVING THE GUARD GATES OF THE NORTHERN CANAL

136. This centre-vent wheel usually operates under about ten feet fall, and is of about sixty horse-power under this fall. It was constructed from nearly the same designs as the model centre-vent wheel, described in art. 100, and represented on plate VII. For a general description of the Guard Gates, see vol. I., page 775, *Appleton's Dictionary of Machines, Mechanics*, etc., New York: D. Appleton & Co., 1852.

A set of experiments upon the power of this wheel was made in 1848, in which the water discharged by the wheel was gauged at a weir constructed for the purpose, below the wheel. The following experiments were made with the same apparatus.

The total length of the weir was 18.02 feet, which, for the purposes of these experiments, was diminished to 16.02 feet by two movable planks or partitions, one foot wide each, the upstream faces of which, when placed upon the weir, were in the same plane as the upstream face of the weir. The form of the weir was such as to give complete contraction; it was constructed of wood with the upstream face vertical. The crest of the weir was formed of southern hard pine plank, four inches in thickness; the top was 0.53 inches wide, and bevelled off on the downstream side, at an angle of 40° with the vertical; the ends of the weir and the sides of the partitions were of the same form.

The bottom of the canal or basin, measured near the weir, was about 6.72 feet below the top of the weir. The water discharged by the wheel passed to the basin through an irregular and contracted channel, cut in rock, and confined by cement masonry. This basin was specially excavated in the rock, of large dimensions, in order that the water might reach the weir in a sufficiently quiet state to permit a satisfactory measurement to be made; and also, for the same object, two gratings were placed across the basin, parallel to each other, and about six feet apart, the downstream grating being about seventeen feet from the weir. The effect of these several precautions was such that, although the water escaped from the wheelpit in a rapid and turbulent current, in the basin between the downstream grating and the weir, the water was tranquil and free from perceptible irregularities in its motion towards the weir.

The depths upon the weir were measured by the hook gauge, described at art. 45 and represented by figures 9, 10 and 11, plate IV.; this was placed in the basin about eight feet from the weir, in a box, in which the communication with the surrounding water was maintained by a small aperture in the bottom; the box and hook gauge were firmly attached to a timber strongly bolted to the masonry forming one side of the basin.

The quantity of water discharged by the wheel is usually regulated by the head gate, admitting the water from the river into the forebay above the wheel. When it is desired to diminish the quantity discharged by the wheel, this gate is partially closed, the effect of which is to diminish the fall acting upon the wheel; but this method was unsuitable for these experiments, on account of the great agitation in the forebay, produced by the fall at the head gate. During these experiments, the head gate was fully opened, and the quantity of water discharged by the wheel was diminished by closing up a portion of the spaces between the guides, with pieces of wood.

The wheel was prevented from revolving by the brake of the Prony dynamometer. The entire apparatus about the wheel remained unchanged throughout the four experiments, except that the head gate was closed on several occasions, to enable the partitions on the weir to be moved. This gate was large (five feet square,) and care was taken to keep it open to its full extent, in all these experiments.

The apertures through which the water entered the wheelpit being the same, the quantity of water discharged must have been uniform, if the head acting upon the orifices had been constant; small variations, however, unavoidably occurred in the head, for which it was necessary to correct the depths upon the weir This has been done in a manner precisely similar to that adopted in the experiments upon the weir at the Tremont Turbine, described a. art. 133.

The apertures in the wheel and between the guides, were entirely submerged The effective height of the water in the wheelpit was measured in a chamber constructed for the purpose, in the masonry. A free communication was maintained between the water in the wheelpit and in the chamber by an iron pipe about 3.5 inches diameter. The surface of the water in the chamber was, in all the experiments, above the level of the top of the apertures between the guides The height above the wheel was taken in the forebay nearly over the wheel, the gauge being placed in a box in the usual manner; the zeros of the gauges, at which both these heights were taken, were at the same level, consequently, the difference in the readings gave the fall acting upon the apertures.

TABLE XI.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE CENTRE-VENT WHEEL FOR MOVING THE GUARD GATES OF THE NORTHERN CANAL AT LOWELL, MASSACHUSETTS

No. of the experiment.	TIME. November 14, 1848.						Length of the experiment, in seconds.	Mean height of the water above the wheel taken in the forebay, in feet.	Mean height of the water below the wheel, taken in the chamber communicating with the wheel pit, in feet.	Fall acting upon the apertures in the wheel and guides, in feet.	Total length of the weirs, in feet.	No. of end contractions in the weir.	Depth on the weir as measured by the hook gauge, in feet.	Constant fall acting upon the apertures in the wheel and guides, to which the depths on the weir are reduced, in feet.	Depth on the weir reduced to the uniform fall in the preceding column, in feet.	REMARKS.
	Beginning.			Ending.												
	H.	min.	sec.	H.	min.	sec.										
1	11	18	2	11	27	42	580	10.373	1.755	8.618	16.02	6	1.0160	8.612	1.0157	Three weirs, 5 34 *feet long* each, separated by partitions one foot wide each.
2	11	52	0	11	58	54	414	10.357	1.745	8.612	16.02	2	1.0072	8 612	1.0072	One weir, both of the partitions placed [illegible] one end.
3	2	1	49	2	16	27	878	10.368	1.746	8.622	16.02	4	1.0137	8.612	1.0133	Two weirs, [illegible] 01 feet long each, separated by the two partitions placed close together, making a single partition two feet wide.
4	2	42	18	2	50	27	489	10.365	1.744	8.621	16.02	2	1.0071	8.612	1 0067	One weir, same [illegible] in experiment 2.

EXPERIMENTS ON THE EFFECT PRODUCED ON THE FLOW OF WATER OVER WEIRS, BY THE HEIGHT OF THE WATER ON THE DOWNSTREAM SIDE

137. These were made at the weir at the centre-vent wheel for moving the guard gates, with the apparatus used in the preceding experiments.

A singular phenomenon was here produced, namely: *under particular circumstances, the flow of water over a weir may be increased by raising the height of the water on the downstream side of the weir* Ordinarily, when water flows over a weir having contraction on the bottom, the under side of the sheet near the weir, is elevated above the level of the top of the weir, taking a curved form; representations of this curve are given in several works, the most perfect of which are by M. M. Poncelet and Lesbros,[1] who ascertained with great care the forms for several depths upon the weir. In such cases, the space between the sheet of water and the plank or other material of which the weir is composed, is filled with air which communicates more or less freely with the external atmosphere.

Suppose the sheet, after passing the weir, to fall into a body of water of considerable depth, in which the natural level of the surface is not very much below the top of the weir, but sufficiently so, as not sensibly to affect the discharge. The weir having complete contraction, the air will remain under the sheet, even if the weir is of very considerable length in proportion to the depth flowing over. Suppose now, that the communication of the air under the sheet, with the external atmosphere, is entirely cut off by placing boards on the downstream side of the weir in contact with each side of the sheet, or by other means, the effect will ordinarily be, that the air under the sheet will be wholly or partially driven out by the lateral communication of motion in fluids, and a partial vacuum will be produced, unless water takes the place of the air that is driven out. In either case, the equilibrium of the atmospheric pressure on the upper and lower sides of the sheet, will be destroyed, the pressure on the upper side preponderating, the effect will be to alter the form of the sheet, and to increase the discharge, by the operation of forces bearing some resemblance to the action in the well-known experiment with Venturi's tube.

In the following experiments, this effect was produced by raising the level of the water on the downstream side of the weir, to a height a little above the top of the weir, in consequence of which, by the lateral communication of motion, the air was driven out, and the flow over the weir facilitated.

---

[1] *Experiences Hydrauliques sur les lois de l'ecoulement*, etc. Paris: 1832. Plate 6.

During the following experiments, the apparatus was arranged in the same manner as in the preceding experiments, with these exceptions, namely: the partitions were not used; the quantity of water entering the wheelpit was diminished by closing up more of the spaces between the guides; the wheel was entirely removed; and means were provided for varying the height of the water on the downstream side of the weir.

The depths on the weir are reduced in the same manner to what they would have been, if the quantity of water entering the wheelpit, and flowing over the weir, had been uniform. The details of the experiments are given in table XII

That the quantity of water entering the wheelpit changed only in a very small degree from any change in the apparatus, is proved by the depths upon the weir in experiments 1 and 9. The circumstances in both being the same, the corrected depth on the weir in experiment 9, is 0.0006 feet less than in experiment 1, corresponding to a change in quantity of about $\frac{1}{910}$ part. A mean of the depths on the weir in these two experiments has been taken, with which to compare the other experiments.

Measurements were also taken of the thickness of the sheet, in the plane of the upstream face of the weir. This was done by means of a graduated rod terminating in a fine point, and so arranged as to slide in a vertical groove, supported from one end of the weir. These measurements were not taken with the same precision as were the depths on the weir with the hook gauge, principally in consequence of the oscillations of the surface.

In consequence of the want of symmetry in the channel carrying off the water from the weir, the water on the downstream side did not assume the same height at both ends of the weir Gauges were placed at both ends, protected in a considerable degree from the agitation of the water immediately below the weir, and placed so as to indicate the height of the water a short distance downstream from the sheet, but the heights taken at these gauges have not the exactness of those taken with the hook gauge. There were also much greater variations in the height of the water during the course of an experiment, than occurred on the upstream side of the weir; some of the heights given in column 9 may consequently be erroneous to the extent of 0.02 feet.

The differences given in column 10, indicate the effect produced on the discharge by the height of the water on the downstream side. When this height was about 3 inches below the top of the weir, the effect was insensible. When about level with the top of the weir, the obstruction was very minute and barely sensible. When the height on the downstream side was about $\frac{3}{4}$ of an inch above the top of the weir, (at which height the air did not remain under the sheet,) the increase in the discharge is quite sensible, the discharge with the same depth being increased about $\frac{1}{10}$ When the height on the downstream side is 1.25 inches above the top of the weir, the obstruction is quite distinct, and it increases rapidly with the increase of height.

TABLE XII.

EXPERIMENTS ON THE EFFECT PRODUCED ON THE FLOW OF WATER OVER WEIRS, BY THE HEIGHT OF THE WATER ON THE DOWNSTREAM SIDE.

1	2						3	4	5	6	7	8	9			10	11	12
No of the experiment.	TIME. November 17, 1848.						Length of the experiment. Seconds	Mean height of the water above the wheel, taken in the forebay	Mean height of the water below the wheel, taken in the chamber communicating with the wheelpit.	Fall acting upon the apertures between the guides.	Depth on the weir as measured by the hook gauge.	Depth on the weir reduced to the uniform fall of 8.242 feet.	Height of the water on the downstream side of the weir, above or below the top of the weir, in feet			Difference of the depth on the weir from the mean in experiments 1 and 9	Actual depth on the weir at the upstream edge of the weir, 3.2 feet from the west end.	REMARKS
	Beginning			Ending.														
	H.	min.	sec	H.	min	sec		Feet.	Feet.	Feet	Feet.	Feet.	East end.	West end.	Mean.	Feet.	Feet.	
1	2	49	0	2	51	30	150	9.762	1.520	8.242	0.8528	0.8528	−1.00	−1.10	−1.050		0.7280	Air under the sheet; form of the sheet not perceptibly changed by raising the water below
2	2	59	0	3	4	30	330	9.753	1.523	8.230	0.8593	0.8527	0.22	0.25	−0.235	+0.0002	0.7247	
3	3	6	30	3	11	30	300	9.753	1.522	8.231	0.8528	0.8532	+0.03	+0.01	+0.020	+0.0007	0.7280	
4	3	18	30	3	24	0	330	9.745	1.519	8.226	0.8480	0.8485	+0.08	+0.05	+0.065	−0.0040	0.7215	No air under the sheet, a very slight lowering of the surface of the water on the downstream side, caused the air to pass under the sheet, and allow it to take the usual form
5	3	29	0	3	34	30	330	9.746	1.525	8.221	0.8515	0.8529	+0.10	+0.07	+0.085	−0.0003	0.7247	No air under the sheet.
6	3	37	30	3	40	0	150	9.738	1.521	8.217	0.8562	0.8571	+0.12	+0.09	+0.105	+0.0046	0.7290	
7	3	45	0	3	49	30	270	9.730	1.528	8.202	0.8806	0.8820	+0.24	+0.20	+0.220	+0.0295	0.7565	
8	3	57	0	4	2	0	300	9.731	1.584	8.147	0.9667	0.9704	+0.50	+0.48	+0.490	+0.1179	0.8607	
9	4	4	0	4	10	[illegible]	360	9.725	1.518	8.207	0.8510	0.8522	−1.06	−1.23	−1.145		0.7260	Air under the sheet, same as in experiment 1.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE LOWER LOCKS, IN LOWELL.

138. In the year 1852, the author, in connection with James F. Baldwin, Esq., the eminent engineer of Boston, Massachusetts, was employed to ascertain the amount of water-power used by the several manufacturing companies at Lowell. In order to be able to do this in a satisfactory manner, it was found necessary to determine anew the rules for computing the discharge of water over weirs of certain forms; and for this purpose an extensive series of experiments was made, with a very complete apparatus, and on a scale of unusual magnitude. The execution of these experiments was intrusted to the author; and *The Proprietors of the Locks and Canals on Merrimack River*, at whose expense they were made, have, with great liberality, given the author permission to publish an account of them.

139. The great difficulty in this kind of experiment, usually, is to obtain a suitable basin in which the water flowing over the weir for a certain period of time may be actually measured. Fortunately for our purpose, the Lower Locks at Lowell are seldom used, except during the high water in the spring, when rafts can pass over the rapids in the river below. These locks were rebuilt principally of wood, in 1841, and at the time when the experiments were made, they were still in good condition; they however required some alterations to adapt them to the requirements of the experiments; which alterations, together with the entire apparatus employed, and the mode of conducting the experiments, will now be described.

140. Plate XI, figure 1, is a general plan of the Lower Locks and the vicinity, on a scale of eighty feet to an inch. *A* is the lower level of Pawtucket Canal; *B*, the Eastern Canal; *C*, the Concord River, which enters the Merrimack River at about 1200 feet below the foot of the lock; *D* is the dam for discharging the surplus water from the Pawtucket Canal into Concord River, passing through the wasteway *E*; *F*, the Middlesex Mills, which are carried by water-power from the Pawtucket Canal, through the covered penstock *H*; *I*, an apparatus erected for the purpose of gauging the water drawn by the Middlesex Mills, which was removed before these experiments were made; *K*, the upper chamber of the lock, which was converted into the gauging basin for these experiments, and which is represented as it was before the alterations were made.

141. Plate XII represents the gauging chamber subsequent to the alterations, on a scale of 10 feet to an inch Figure 1 is a plan; figure 2, a longitudinal section; and figures 3 and 4 transverse sections. The side wall *A* was built in 1822, of large and small stones laid without mortar; in order to render the lock capable of holding water, it is lined with planks about three inches thick, secured by tree-nails and spikes to wooden frames, which are supported on the bottom by the earth and some rough walls, and on the sides by the side walls of the lock. As originally constructed, the planking was fastened to posts resting immediately against the side walls; but when reconstructed in 1841, the chambers, together with the gates, were narrowed to the width represented, which is about one half the former width; at that time, also, the parts *B B*, about the hollow quoins, were built anew in cut granite, laid in hydraulic cement.

To prepare the chamber for these experiments, the upper set of lock gates and the corresponding mitre sills were removed, and the weir *C*, plate XII., figures 1 and 2, constructed in place of them; the middle gates were also removed, and the lower end of the chamber closed with timbers and plank, as represented at *D*; in the lower part of this timber work the waste gate *K* was constructed, for the purpose of drawing off the water from the chamber, after each experiment.

The construction of the wooden sides of the chamber was such, that when the chamber was partially or wholly filled with water, they would yield a little to the pressure, and the capacity would, consequently, be increased beyond what it was when empty, which was necessarily the case when the dimensions were taken. To diminish, as much as practicable, this source of error, the braces *E* were placed across the chamber, just above the water-line *F F*, nearly up to which the chamber was filled in the experiments. These braces were placed opposite each side timber in the frame of the chamber, excepting at *G G*, where a flooring of thick plank, put in for another object, answered the same purpose; afterwards, every accessible timber in the sides was strongly braced and keyed up from the side walls, which was done with such force, that the ends of the braces *E* were indented into the planks forming the sides of the chamber. At *H H*, where the space between the walls and the planking was too small to admit of the bracing, the spaces between the timbers were filled with small stones, dropped and rammed in from the top. These operations stiffened the sides of the chamber so much, that the correction required for the enlargement of the capacity of the chamber, in consequence of the yielding of the sides, was very minute. All the leakages that could be detected were stopped by various contrivances; the depressions in the planks, about the heads of the spikes, were filled up with cement; the sides of the planking towards the chamber were scraped, and rendered as smooth and uniform as practicable.

A part of the wall *A* was removed at *I*, for the purpose of discharging the water, flowing over the weir, directly into the wasteway, whenever it was necessary to divert its flow from the chamber; the floor *GG* was continued through the wall, as represented in figures 1 and 4, plate XII.

142. Plate XIII., figure 1, is a longitudinal sectional elevation through the middle of the weir, showing most of the apparatus immediately connected with it. *A* is a plate of cast-iron forming the crest of the weir; it is ten feet long, thirteen inches wide, and an inch thick, accurately and smoothly planed in every part; the upper corner presented to the current is square and sharp, or as nearly so as cast-iron can be conveniently maintained; the horizontal part of the top is 0.25 inches wide; the remainder of the top is bevelled off at an angle of 45°; this plate is secured to the timber work by numerous screws with countersunk heads; the timber work is strongly bolted to the granite hollow quoins of the lock. The ends of the weir *B* are formed of plates of cast-iron, of similar section to the plate *A*. The whole upstream side of the weir forms a vertical plane 13.96 feet in length, and 4.60 feet in depth, from the top of the plate *A* to the top of the masonry *C*; the upstream side of the plates *B* are also in the same vertical plane.

D is the swing gate for admitting and diverting, at will, the stream of water flowing over the weir, into or from the measuring chamber *E*. *FFF* are leak boxes or troughs, to catch the leakage by the edges of the swing gate, when shut. The water thus caught is conveyed to openings *G*, cut through the planking on each side of the chamber, through which it is discharged, thus preventing any embarrassment from the leakage of the swing gate when shut, as it does not enter the chamber *E*. The swing gate is suspended from the pivots *H*; all its parts are made as light as practicable, consistent with the required stiffness, in order that the time occupied in opening or shutting it may be as short as possible. A very important part of the experiments consisted in determining the length of time during which the water flowed into the measuring chamber *E*; this was obtained by observing the time when the swing gate was opened and shut, which was done by an observer in the building *I*, by means of an electric telegraph and a marine chronometer, in the following manner. The break circuit apparatus *K* is fixed in such a position that, when one half only of the stream flowing over the weir passes into the chamber, the cam *L*, attached to the frame of the swing gate, depresses the knob as represented in the plate, and breaks the circuit of the electric current in the wire *M*; this causes a sound to be made by the call *N*, in the small building *I*, where sits the observer with his eye on the chronometer, who notes the time when the sound is made; the chronometer used beats half seconds, but, by employing [illegible] practised observer, the time was noted to tenths of a second, the error probably rarely exceeding two tenths of a second. The gate, with its accompanying apparatus, was balanced, so that it could be opened or shut with sensibly the same amount of force; this balancing was done with the water flowing over the weir, and was done anew for each material variation in the quantity. To each of the timbers *O* and *P*, plate XIII, figure 1, was attached, by a joint, a prop *L*, shown at figure 4, plate XII; the prop at the timber *O*, for the purpose of retaining the swing gate in its position when open, and the other at the timber *L*, to retain it in position when shut. The movement of the gate was produced by placing weights upon the frame at *Q* and *R*, plate XIII., where the gate is represented [illegible] at the middle point of its motion while shutting; the motion being produced by the gravitation of the weights at *Q*. As soon as the gate is shut, the prop *L*, plate XII., figure 4, is placed under the frame at *R*, plate XIII., and keyed up tight; the weights are then taken off at *Q*, and about the same amount of weight is placed at *R*; then, when it is desired to open the gate, an assistant strikes the prop from under *R* with a sledge-hammer, when the weight at *R* causes the gate to open; the prop is then immediately placed under the frame at *Q*. To prevent injurious concussions from the action of the weights, thick pieces of India-rubber, operating as springs, were fastened on the under-side of the frame at *Q* and *R*, which, when the gate attained either of its extreme positions, struck upon the corresponding stops *S* and *T*.

From the foregoing description of the apparatus, the manner of operating the swing gate will be readily understood. Four assistants were employed for the purpose. Suppose that the chamber is nearly filled, and that it is required that, when the water reaches a certain height, the flow of the water shall be diverted from the chamber; one assistant, who has been watching the rise of the water, gives [illegible] signal when the water has reached the desired height, at which the prop under the frame at *Q* is immediately knocked away, the weights at *Q* cause the gate to move until it strikes the weir, or the India-rubber springs strike the stops *T*; at that moment another assistant places the prop under *R*, and the flow of the water [illegible] diverted from the chamber; another assistant then changes the weights, and the apparatus is ready for the reverse operation by which the gate is opened. Much time was occupied in adjusting this apparatus so that the cam *L*, plate XIII., would strike the break circuit when the gate was in such a position that one half of the water flowing over the weir passed into the chamber; and also, that the time in which the gate moved through each half of the thickness of the sheet, would be the same It required a new adjustment for each depth upon the weir Precise accuracy was not attained or attempted, in any of these adjustments, but such an approximation was made, that it is believed that the errors arising from want of complete exactness, are entirely insensible in the results.

148. The depths upon the weir were observed by means of the hook gauges *U* and *V*, plate XII, figures 1 and 2, and plate XIII., figure 1. One of these gauges is represented in detail by figures 2, 3, and 4, plate XIII., ½ the full size. They were made by the Lowell Machine Shop. This valuable instrument has been sufficiently described in the account of the experiments on the Tremont Turbine (art 45). These gauges were placed in wooden boxes closed on all sides, excepting at the top; in the bottom of each of which was a hole about an inch in diameter, and in that part of the bottom projecting beyond the lines of the canal walls, due care being taken that the plugs, by which the holes were partially closed, did not project through the bottom. In the experiments on the weir in which the end contraction was suppressed, a communication was established between the gauge boxes and the canal leading to the weir, by pipes opening at *B*, figures 8, 9, and 10, plate XIV. The pipes opening near the bottom of the canal, six feet from the weir, forming part of the system for taking the heights at different distances from the weir, were also used in some of the experiments. The boxes were securely fastened to wooden posts in the angles of the gate recesses; and the posts were strongly fastened to the walls, by several iron bolts driven into holes drilled in the granite stones for the purpose. It was very important that these gauges should be immovably fixed, relatively to the weir. It is probable, however, that they were not perfectly firm. During the course of the experiments, two comparisons were made of the relative heights of the gauges and the top of the weir; one on October 26th; the other, November 8th, when there was found to be a sensible difference in them, the most probable cause of which was, that changes took place in the absolute height of the gauges, that did not affect the weir in the same degree. It is difficult to perceive how the weir could change from a settlement of the masonry, founded, as it is, on rock; the walls to which the gauges were attached, were much less substantially built and not founded on rock; it is not impossible that changes took place in the timber-work of the weir, by the absorption of water, notwithstanding it was

fixed in place several weeks before the experiments were made, with a view to its complete saturation.

If the apparatus is sufficiently stable, the comparison of the heights of the hook gauges with the top of the weir, can be made with any desired degree of precision. For making the comparison in these experiments, the following apparatus was devised. The water being drawn out of the canal, the top of the weir was inclosed in a water-tight trough, containing only a small quantity of water, but sufficient to cover the crest of the weir to a small depth; this trough was connected with the hook gauge boxes, by leaden pipes; the boxes were rendered water-tight by coating the joints with pitch, and plugging up the holes in the bottom; they were also carefully propped up. The communication being free, and the leakages very small, the water on the crest of the weir and in the boxes, would stand at the same level; consequently, all that remained to be done, was to measure the height of the water with the hook gauges, and, at the same time, the depths upon the crest of the weir. The measurement by the hook gauges presented no difficulty, as it required nothing more than the ordinary use of the instruments. To measure the depths upon the crest of the weir, had always been a difficulty in making similar comparisons; to meet it in this case, the instrument, represented at plate XIII, figure 5, was devised. The points were numbered from 1 to 10, and the exact height of each of them above a horizontal plane, on which the instrument stood, was ascertained. In using this instrument, the water in the trough was adjusted to a convenient level; the top of the weir was divided into ten equal spaces; the instrument was placed upon one of them, and when the water became quite tranquil, the number of the point that coincided with the surface was noted, and, at the same moment, the heights of the water in the boxes were observed with the hook gauges. If (as was usually the case) the surface of the water did not exactly coincide with either of the points, the true fractional number was taken by estimation. The adjacent points differed in height about 0.001 feet, and a fourth part of this quantity was sufficiently distinct not to be doubtful.

As an example of the precision attainable by the use of this instrument, the following results are given of the comparison of the north hook gauge with the weir, made during the night of October 20, by Mr. John Newell. The results indicate the corrections to be applied to the reading of the hook gauge, to give the true height of the surface of the water in the gauge box, above the top of the weir, each result being a mean of eleven measurements made at equidistant points on the weir.

By the 1st trial, the correction was . . . . −0.03070 feet.
" 2d " " " . . . . −0.03032 "
" 3rd " " " . . . . −0.03076 "
" 4th " " " . . . . −0.03096 "
" 5th " " " . . . . −0.03079 "
Mean . . . . −0.03072 feet.

The extreme variation is between the 2nd and 4th trials, amounting to 0.00064 feet, a quantity scarcely visible to the naked eye; of course, in the mean result of all the trials, the error of observation must be entirely insensible.

It has been remarked that the comparisons made at different times, did not give the same results. Two complete comparisons were made, as follows:—

DATE, 1854	CORRECTIONS	
	North hook [illegible] Feet	South hook gauge Feet
October 26th	−0.03072	−0.02786
November 8th.	−0.03250	−0.03060

Considering the care with which these comparisons were made, and the perfection of the method, the differences cannot be attributed to errors of observation, but, rather, to a want of stability in some parts of the apparatus. The corrections determined October 26th, were used in reducing all the experiments made from October 20th, to November 7th, both inclusive; for all subsequent experiments, the corrections found November 8th were used.

The twenty-three experiments numbered from 11 to 33, in table XIII, were made under circumstances as nearly identical as practicable. They were made at different times throughout the course, for the purpose of neutralizing errors of the same class as that just described, the resulting effects of which ought to be shown by the variation in the coefficients deduced from experiments made at different times. These experiments are collected together in the following table:—

DATE 1852	Number of experiments	Mean coefficients	Differences from the mean deduced from all the experiments or from 3.3223
Oct. 20th, P. M., and Oct. 21st, A. M.	6	3.3186	−0.0037
Oct. 21st, P. M., and Oct. 22d, A. M.	8	3.3216	−0.0007
Oct. 29th, P. M.	[illegible]	3.3278	+0.0055
Nov. 11th, P. M.	[illegible]	3.3207	−0.0016

The extreme variation is between the experiments of October 20th and 29th, in which it amounts to $\frac{1}{361}$. The greatest difference from the mean deduced from all the 23 experiments, is in the coefficient deduced from the experiments of October 29th, in which it amounts to $\frac{1}{367}$. It is fair to presume that similar irregularities, not in any case much exceeding the above, and arising principally from want of stability in the apparatus, exist in other parts of this series of experiments.

144. The capacity of the gauging chamber was obtained by measuring its dimensions. For this purpose, horizontal lines were traced on the sides of the chamber at every foot in height; the widths were then measured at right angles to the sides, at points two feet apart; from these widths, and other necessary measurements, the total area was obtained at each horizontal section. When these measurements were made, the chamber was of course empty, but when filled with water, its dimensions would evidently be somewhat larger, in consequence of the sides and bottom yielding to the pressure. To ascertain what allowance to make for this, a systematic measurement was made in the spaces between the planking and the walls, both when the chamber was empty and when filled to the usual height; similar measurements were made for the bottom, by placing poles vertically, resting upon, and fastened to the bottom; the elevations of the tops of these poles were taken with a levelling instrument, both when the chamber was empty, and when filled. It was thus ascertained that the capacity of the chamber, when filled with water to the usual height, was 11.11 cubic feet greater than when empty.

Two persons made independent measurements of the capacity of the chamber, the results of which differed only about ¼ of a cubic foot, a coincidence which must of course be considered as accidental. The capacity finally determined upon for 9.5 feet in height, (which was nearly the depth filled in each experiment,) and including the enlargement resulting from the pressure, was 12138.18 cubic feet.

145. The chamber was not quite water-tight, but the amount of the leakage was determined by noting the rate at which the surface of the water lowered, when none was admitted from the weir, and the waste gate was closed; this was repeated with the water in the chamber at different depths. It was thus found that the mean leakage was 0.085 cubic feet per second; that is, the product of 0.085 multiplied by the number of seconds that the water flowing over the weir continued to enter the chamber during an experiment, must be added to the quantity in the chamber at the moment the water was diverted, in order to give the true quantity that passed over the weir in the same time.

146. It was not convenient to empty the chamber entirely after each experiment, but the heights of the water in the chamber at the beginning and ending of each, were ascertained with great accuracy by means of hook gauges, placed in the boxes *X* and *Y*, figures 1, 2, and 3, plate XII., which were fastened to a post strongly bolted to the wall *A*. A communication was established, at will, between the water in the chamber and either of the boxes, by pipes and cocks. The operation of taking the heights was as follows: the chamber having been sufficiently emptied, the waste gate *K* was closed, the communication of the lower box with the chamber was established, and when the oscillations in the surface had ceased, the height of the water was taken; the cock was then shut, and a signal made for opening the swing gate. When the chamber had been filled, and the flow of water into the chamber diverted by closing the swing gate, the communication with the upper box was opened; when the oscillations had ceased, observations of the water were taken at short and regular intervals, for some minutes, the time and height being noted. In consequence of the leakage of the chamber, the surface lowered slowly, and the continued observations were made for the purpose of being able to infer the exact height at which the water stood in the chamber at the instant that the swing gate was shut, the very slow rate at which the surface of the water in the chamber lowered, permitting this to be done with great precision. For the success, however, of this operation, it was essential that the timekeeper used should agree with the chronometer, by which the times of opening and shutting the swing gate were noted; it was accordingly frequently compared, and any difference noted.

147. Plate XIV. represents the different forms of weir on which experiments were made. All the figures are on the same scale, namely, five feet to an inch, or $\frac{1}{60}$ the full size.

Figure 1 is a longitudinal section, figure 2, a plan, and figure 3, an elevation of what we call the regular weir, that is, a weir in which the contraction is complete, both on the ends and on the bottom

Figure 4 is an elevation of a weir of precisely the same form as that last described, excepting that it is divided into two equal parts or bays by the partition, which is two feet wide. The upstream side of the partition is in the same vertical plane as the remainder of the weir, having no bolt heads or other projection below the level of the surface of the water

Figures 5, 6, and 7, represent a weir of precisely the same form as that first above described, excepting that the depth of the canal approaching the weir is diminished

Figures 8, 9, and 10, represent the same weir as first above described, modified so that the contraction at the ends is suppressed, that is, the canal leading to the weir is of the same width as the weir. These figures also show the apparatus used to ascertain the effect of taking the depths upon the weir at different distances from it, by means of pipes opening near the bottom of the canal.

Figures 11 and 12 represent the upper part of a dam, of the same section as that erected by the Essex Company, in 1846–8, across the Merrimack River at Lawrence, (about nine miles below Lowell). This magnificent work has an overfall 900 feet in length, the perpendicular fall being about 24 feet. This form was experimented upon, in order to obtain a formula for computing the flow of the river over this dam.

DESCRIPTION OF TABLE XIII.

Containing the details of the experiments on the flow of water over weirs, made at the Lower Locks, Lowell, in October and November, 1852.

148. The columns numbered from 1 to 5, require no further explanation than is contained in the respective headings.

149. COLUMN 6. *Duration of the experiment.* This is the interval of time during which the water flowed into the chamber; it is obtained by taking the difference of the corresponding times in column 5.

150. COLUMN 7. *Mean depth upon the weir by observation.* It was found impracticable in many cases, to maintain the canal at a uniform height throughout an experiment, although every endeavor was made. For instance, no experiments were made when the mills were in operation, nor until some hours after the usual time when they ceased drawing water; this rendered it necessary to perform the experiments either during the night, or on Sunday; in consequence of the lateness of the season, advantage was taken of both these opportunities. When any change was made in the level of the water in the canal, for the purpose of varying the depths upon the weir, a considerable time was allowed to elapse before the experiments were resumed, in order that the level of the water might get well established. In spite of all precautions, however, variations frequently occurred in the depths upon the weir, which, with the ordinary mode of taking an arithmetical mean of the several observations of the depth, would have materially affected the accuracy of the results; this difficulty was obviated in a

great degree, by the use of a novel mode of obtaining the mean depth, which will now be explained. Let

h, h', h'', etc. h^n represent the several observed depths upon the weir, the successive values not differing greatly from each other.

$t, t'\ t''$, etc. t^n, the corresponding intervals of time between the several observations;

T, the sum of all the intervals of time,

Q, the total volume of water actually flowing over the weir in the time T;

H, the mean depth upon the weir that would discharge the volume Q, in the time T;

l, the length of the weir;

C, a constant coefficient:

we shall have, evidently, *very nearly*,

$$Q=\frac{t}{2}Clh^{\frac{3}{2}}+\frac{t+t'}{2}Clh'^{\frac{3}{2}}+\frac{t'+t''}{2}Clh''^{\frac{3}{2}}+\text{etc.}+\frac{t^n}{2}Clh^{n\frac{3}{2}}:$$

$$Q=Cl\left(\frac{t}{2}h^{\frac{3}{2}}+\frac{t+t'}{2}h'^{\frac{3}{2}}+\frac{t'+t''}{2}h''^{\frac{3}{2}}+\text{etc.}+\frac{t^n}{2}h^{n\frac{3}{2}}\right):$$

we have also

$$Q=TClH^{\frac{3}{2}}:$$

whence we derive, by substituting the value of Q previously found,

$$H=\left\{\frac{\frac{t}{2}h^{\frac{3}{2}}+\frac{t+t'}{2}h'^{\frac{3}{2}}+\frac{t'+t''}{2}h''^{\frac{3}{2}}+\text{etc}+\frac{t^n}{2}h^{n\frac{3}{2}}}{T}\right\}^{\frac{2}{3}}.$$

As an example of the application of this method, let us take the observations made at the north hook gauge during experiment 74; this is selected, simply because the variations in the depths upon the weir were greater than in any other experiment.

EXTRACT FROM THE NOTES TAKEN AT THE NORTH HOOK GAUGE

	October 21, 1852, A. M., 9h 5', watch 12" fast.		
	TIME	Reading of the hook gauge	
	9h 9' 15"	0.6300	
	10 50	0.6320	
Commencement of the experiment by the time of *this watch* 9h 12' 12.4"	11 45	0.6325	
	12 45	0.6310	1
	13 15	0.6310	1
	14 20	0.6300	1
	14 50	0.6365	3
	15 20	0.6290	1
	16 30	0.6300	1
	17 5	0.6335	2
	17 55	0.6380	3
	18 35	0.6480	5
	19 20	0.6500	6
	20 0	0.6470	5
	20 55	0.6470	5
	21 25	0.6415	4
	22 10	0.6540	7
	22 35	0.6550	7
Ending of the experiment by the time of *this watch* [illegible] 21' 49.9"	23 5	0.6480	5
	23 45	0.6580	8
	24 35	0.6605	9
	Arithmetical mean reading,	0.6428	

For the purpose of simplifying the operation of finding the mean, it is assumed that we can, without sensible error, use an arithmetical mean of all depths not varying more than 0.002 feet from each other; accordingly an arithmetical mean has been taken of all the readings marked 1 in the margin of the above table, and similar means have been taken of the other readings marked with the same number in the margin. It will be perceived that it was noted at 9h 5', that the watch was 12" fast; by another comparison with the chronometer made at 10h 47', the watch was 22" fast; from these two comparisons it is inferred that, at the middle of the experiment, the watch was 13.3" fast. Instead of changing the times of all the observations, the time of the commencement and ending of the experiment has been changed to conform to this watch, but for the purpose of this reduction only. By the method adopted, it is assumed that the height of the water did not change until half the interval of time between two consecutive observations had elapsed; accordingly, we find that the time corresponding to the first mean depth, is from the beginning of the experiment to 9h 14′ 35″, or 142.6″, and from 9h 15′ 5″ to 9h 16′ 47.5″ or 102.5″, making 245.1″. The several mean readings and the corresponding times, given in the following table, are obtained in this manner; the depths upon the weir corresponding to the several mean readings, are also given, which are found by subtracting 0.03072 foot from each mean reading, (see art. 143).

Number of the mean reading	Mean reading of the hook gauge Foot	Time corresponding to each mean reading Seconds	Mean depths upon the weir, deduced from the several mean readings Feet
1	0.63020	245.1	0.59948
[illegible]	0.63350	42.5	0.60278
3	0.63725	75.0	0.60653
4	0.64450	37.5	0.61378
5	0.64750	167.5	0.61678
6	0.65000	42.5	0.61928
7	0.65400	62.5	0.62328
[illegible]	0.65800	45.0	0.62728
9	0.66050	39.9	0.62978

The quantities in the third column of this table are the values of $\frac{t}{2}$, $\frac{t+t'}{2}$, etc., in the expression given above for H; the quantities in the fourth column are the corresponding values of h, h', etc. The value of T being 757.5, all the quantities in the second member of the equation are known; by substituting these values we find

$$H = 0.6113.$$

The arithmetical mean of the eighteen observations is 0.6428; deducting the correction 0.03072, we find the mean depth to be 0.6121; the difference by the methods is 0.0008.

A similar computation on the observations at the south hook gauge gives

$$H = 0.6099.$$

By taking the arithmetical mean of the observations, we find the depth, by the south hook gauge to be 0.6096.

The mean of the above values of H, or 0.6106, is adopted as the depth on the weir in experiment 74.

A similar reduction has been made of the observations at each hook gauge, in all the experiments; the arithmetical mean of the two results obtained for each experiment, is given in column 7.

Notwithstanding the advantage attending this mode of reduction, it cannot be denied that, for the most perfect experiments, the depth on the weir should be invariable throughout, and that, *cæteris paribus*, the experiments will be the less valuable, the greater the variation. To enable the reader to judge of the relative value of the experiments, as far [illegible] it depends upon this variation, the small figures to the left and above the several depths in column 7 are given; they indicate the highest number of values of h, h', h'', etc. used in the reduction of the observations, at either of the hook gauges, in the corresponding experiments.

151. Column 8. *Mean velocity of the water approaching the weir.* This is obtained by dividing the corresponding quantity of water flowing over the weir, given in column 14, by the area of the section of the canal, at the hook gauge boxes. In the weir having contraction at the ends, this would strictly include all the space under the gauge boxes, although, from the form of the walls, it is evident that the current could flow only in a small part of this space; consequently, the portion in which the current could not flow is not included in the areas used.

152 Column 9 *Head due to the velocity in column 8.* This is sufficiently explained in the heading.

153. Column 10. *Depth upon the weir, corrected for the velocity of the water approaching the weir.* In the common formula for the discharge of water over weirs,

$$Q = ClH\tfrac{2}{3}\sqrt{2gH}. \qquad (A)$$

The second member may be separated into three factors, namely: C, the coefficient of contraction; l, the length of the weir; and $H\tfrac{2}{3}\sqrt{2gH}$, the theoretical discharge for the unit of length. According to a well-known elementary theorem in hydraulics, the latter factor may be represented by the area of a segment of a parabola, of which the parameter is $2g$; thus, in figure 5, plate XII, if $AB = H$, and $BC = \sqrt{2gH}$, and the curve AMC is a parabola, of which the vertex is A, we shall have the area of the segment $ABC = H\tfrac{2}{3}\sqrt{2gH}$; also, the velocity of the fluid at any point P will be represented by the ordinate PM. The factor $H\tfrac{2}{3}\sqrt{2gH}$ may also be decomposed into two others: $H = AB$, and $\tfrac{2}{3}\sqrt{2gH}$, which equals the mean value of all the ordinates of the parabola between A and C, and represents the mean velocity of the fluid for the whole height of the orifice. In demonstrating this theorem, it is assumed that the water in the reservoir is at rest; we can, however, easily establish an analogous theorem, in which it is assumed that the water in the reservoir has [illegible] velocity approaching the weir, in the direction perpendicular to the plane of the weir. Suppose h to be the head due this velocity; and in figure 6, plate XII, let $AB = H$, and $AD = h$, we shall have for the velocity v', at any point P in the height of the orifice,

$$v' = \sqrt{2g(AP + h)}:$$

but this value of v' is the ordinate corresponding to the abscissa, $AP + h = DP$, of a parabola whose parameter is $2g$. We have also

$$BC = \sqrt{2g(H + h)}.$$

We can, consequently, represent the discharge for the unit of length, by the area of the surface $ABCG$, which is a portion of the segment BCD; the area of $ABCG$ is the difference of the areas of the segments BCD and AGD; the area of BCD is

$$\tfrac{2}{3} BD \times BC = \tfrac{2}{3}(H + h)\sqrt{2g(H + h)},$$

and the area of ADG is

$$\tfrac{2}{3} AD \times AG = \tfrac{2}{3} h\sqrt{2gh},$$

consequently, the area of $ABCG$ is

$$\tfrac{2}{3}(H + h)\sqrt{2g(H + h)} - \tfrac{2}{3} h\sqrt{2gh} = \tfrac{2}{3}\sqrt{2g}\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right];$$

and for the total discharge we have

$$Q' = Cl\tfrac{2}{3}\sqrt{2g}\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right]. \qquad (B)$$

The formula (A) may be put under the form

$$Q = Cl\tfrac{2}{3}\sqrt{2g}\,H^{\frac{3}{2}}. \qquad (C)$$

Suppose H' to represent a depth upon the weir that would give the discharge Q' by the formula (C), we shall have

$$Q' = Cl\tfrac{2}{3}\sqrt{2g}\,H'^{\frac{3}{2}}:$$

substituting the value of Q' in (B), and reducing, we find

$$H' = \left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right]^{\frac{2}{3}}. \qquad (D)$$

The equation (B), from which this value of H' is derived, does not agree with that given for a similar case by most writers on hydraulics, who seem generally to have followed Du Buat;[*] it agrees, however, with the expression given by Weisbach,[†] who appears to have been the first to point out the error.

The formula (D) was communicated to the author, in 1849, by Mr. Boydon, accompanied by a demonstration somewhat resembling the above.

The values of H', given in column 10, have been computed by the formula (D) from the corresponding values of H and h in columns 7 and 9.

154. Columns 11, 12, and 13 are sufficiently explained by their respective headings.

155. Column 14. *Quantity of water passing the weir per second.* The quantities in this column are obtained by dividing the total quantities given in column 13, by the corresponding intervals of time in column 6.

156. Column 15 *Value of C in the formula*

$$Q = C(l - 0.1 n H') H'^{\frac{3}{2}},$$

Q having the corresponding values in column 14.

In the formula proposed at art. 124, namely:—

$$Q = C(l - bnh)h^a,$$

the values of the constants [illegible] and b are to be determined by experiment. The values adopted in the formula by which the coefficients in this column have been computed, namely: $a = \frac{3}{2}$, $b = 0.1$, were determined upon after many trials of other values; in consequence of their giving results according the most nearly with all the experiments, and at the same time having a convenient degree of simplicity. It is quite likely that many other values of a and b (probably an unlimited number) might be found that would accord somewhat nearer with the experiments; a closer approximation than is given by the use of the values adopted, could have, however, but little practical value; much less, it was thought, than would be derived from the use of the simple values adopted. The use of a fractional power, such as $a = 1.47$, deduced from the experiments at the Tremont Turbine (art. 135), is very inconvenient, and, to persons not well skilled in the use of logarithms, offers great difficulty.

---

[*] *Principes d'Hydraulique, etc., by M. Du Buat.* Paris: 1816. Vol. 1, page 201.

[†] *Allgemeine Maschinen Encyclopadie.* Leipzig: 1841. Vol. 1, page 489.

157. Columns 16, 17, and 18, are, for the purpose of obtaining correct mean results of the experiments, made under circumstances nearly identical. In consequence of the variations in the height of the canal (art. 150), it was impracticable to repeat the experiments with precisely the same depth upon the weir; by the method adopted for obtaining these mean results, all inconvenience from this source is obviated. As the formula by which the values of C, in column 15, are obtained, is such as to give results agreeing very nearly with experiment, even when the depths differ considerably, it is plain that the values of C deduced from experiments having nearly the same depths, cannot be affected by small variations in the depths, and will be subject to no greater irregularities than if, in the several experiments from which they are deduced, the depths had been precisely the same. We can consequently take a mean coefficient with the same confidence that we could take a mean quantity, if the depths had been precisely the same. These mean coefficients are given in column 16. In column 17 are given depths on the weir, nearly a mean of those in the experiments from which the corresponding mean coefficients have been deduced. In column 18, are given what may be called the mean quantities of water actually found by experiment to be discharged with the corresponding depths in column 17. A method similar to the above was used to reduce the quantities discharged in the experiments of Castel, reported in the *Annales de Chimie et de Physique*, vol 62. Paris: 1836; reprinted in the first volume of the *Annales des Ponts et Chaussées* for 1837.

158. Column 19. *Quantity of water passing the weir, calculated by the formula*

$$Q = 3.33\,(l - 0.1\,n H'')\,H''^{\frac{3}{2}},$$

H'' *having the corresponding values in column* 17.

The coefficient 3.33 is derived from the arithmetical mean of all the coefficients in column 15, which is 3.3318, the two final decimals being omitted for the sake of simplicity. The largest coefficient in column 15, is that deduced from experiment 34, which is 3.3617, exceeding the coefficient adopted by $\frac{1}{105}$ part; the smallest coefficient is that deduced from experiment 4, which is 3.3002, being less than the coefficient adopted, by $\frac{1}{112}$ part; that is, the formula by which the quantities in column 19 are computed, will represent every experiment in the table, within one per cent.

159. Column 20. *Proportional difference, or the absolute difference of the quantities in columns* 18 *and* 19, *divided by the quantity in column* 18. The greatest proportional difference is that deduced from experiments 34 and 35, which is —0.0090, or a little less than one per cent. In these experiments there were two weirs,

about four feet long each, separated by a partition two feet wide; the near neighborhood of the two orifices appears to have affected the discharge. The next largest proportional difference is that deduced from experiments 36 to 43, which is —0.0068, or about $\frac{2}{3}$ of one per cent.; in these experiments, the depth of the water in the canal leading to the weir, was only about three times the depth upon the weir. The experiments with the diminished depth in the canal were made for the purpose of testing the method of correcting the depths, upon the weir, for the velocity of the water approaching the weir (art. 153). They indicate that the method is not strictly accurate, as might have been anticipated, omitting, as it does, all consideration of the effect produced by this velocity, in modifying the contraction. It is well understood that such an effect is produced,* but it is of such a complicated nature, that the investigations hitherto undertaken have thrown but little light upon it.

It will be perceived by referring to column 4, that the experiments 51 to 55 were made under the same circumstances as experiments 44 to 50, excepting that the sheet of water, after passing the weir, was prevented from expanding laterally for a certain distance. This was accomplished by placing boards at the ends of the sheet, as represented by the broken lines at *A*, figures 8 and 9, plate XIV. By referring to column 16, it will be seen that the effect of these boards was to diminish the coefficient from 3.3409 to 3.3270, corresponding to a diminution of the quantity discharged by the weir, with the same depth, of $\frac{1}{240}$, or about four-tenths of one per cent.; in other words, the effect of the boards upon the discharge was the same as would be produced by shortening the weir $\frac{1}{480}$, or $\frac{1}{4}$ inch, at each end. By reference to figure 8, plate XIV., it will be perceived that these boards did not affect the free communication between the atmosphere and the air under the sheet of water; if this communication had been obstructed, so that the pressure of the air under the sheet had been different from that of the atmosphere, it would have affected the discharge.

---

* *Jaugeage des cours d'eau, etc., by M. P. Boileau*, page 40. Paris: 1850.

TABLE

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE

1	2	3		4	[illegible]						6	7	8
Number of the experiment.	Date of the experiment. [illegible]	Temperatures by Fahrenheit's thermometer.		Reference to the figures on plate XIV, and particular description of the weir.	Time of the commencement and conclusion of the experiment, as indicated by the telegraph signals.						Duration of the experiment.	Mean depth upon the weir by observation. H	Mean velocity of the water in the canal at the transverse section at the hook gauge boxes, or 6 feet from the weir. V
		Of the air in the shade.	Of the water.		Commencement.			Conclusion.					
					h.	m.	sec.	h.	m.	[illegible]	[illegible]	Feet.	Feet.
1	Oct. 27, P.M.			*Figures 1, 2, and 3.* Width of the canal on the upstream side of the weir 13.96 feet. Mean depth of the canal opposite the hook gauge boxes 5.048 feet below the top of the weir.	10	15	0.8	10	18	13.6	192.8	^{2}1.52430	0.7782
2	" " "				11	20	1.3	11	23	14.2	192.9	^{2}1.55015	0.7813
3	" " "				11	54	1.3	11	57	10.6	189.3	^{2}1.55030	0.7882
4	" 28, A.M.	48.75°	46.5°		0	26	17.9	0	29	20.1	182.2	^{1}1.56010	0.7889
5	Oct. 24, P.M.	52°	48.5°	*Figures 1, 2, and 3.* Same as the preceding.	9	4	0.5	[illegible]	8	20.5	260.0	^{4}1.24690	0.5001
6	" " "				9	33	2.3	[illegible]	37	21.5	259.2	^{2}1.24195	0.5033
7	" " "				10	0	1.7	10	4	18.4	256.7	^{4}1.24795	0.4971
[illegible]	" [illegible] "				10	31	2.1	10	35	20.6	258.5	^{4}1.25085	0.5041
9	" " "				11	0	2.3	11	4	16.1	253.8	^{4}1.25290	0.5000
10	" [illegible] "				11	30	1.8	11	34	21.4	259.6	^{4}1.25490	0.5087
11	Oct. 20, P.M.			*Figures 1, 2, and 3.* Same as the preceding.	10	1	0.8	10	7	22.0	381.2	^{8}0.96711	0.4256
12	" " "				10	30	0.9	10	35	41.0	340.1	^{8}1.02755	0.4591
13	" [illegible] "				11	12	0.7	11	17	47.4	346.7	^{8}1.03395	0.4629
14	" [illegible] "				11	48	0.6	11	53	43.3	342.7	^{8}1.03815	0.4611
15	" 21, A.M.				0	24	59.5	0	30	37.5	338.0	^{8}1.04060	0.4680
16	[illegible] " "	48°	49°		1	0	0.0	1	5	37.4	337.4	^{8}1.03785	0.4666
17	[illegible] " P.M.				[illegible]	48	8.0	9	54	36.4	388.4	^{8}0.96825	0.4233
18	" " "				10	23	1.2	10	29	9.9	368.7	^{8}0.97530	0.4301
19	" " "	42°	48.75°		10	52	0.4	10	58	8.7	368.3	^{8}0.97950	0.4318
20	" " "				11	23	1.4	11	29	4.2	362.8	^{8}0.98885	0.4377
21	" " "				11	53	1.5	11	59	3.5	362.0	^{8}0.99460	0.4418
22	" 22, A.M.				0	43	0.0	0	49	48.8	408.8	^{8}0.91570	0.3951
23	[illegible] " "	42°			1	12	0.7	1	18	40.9	400.2	^{8}0.92800	0.4015
24	" " "				1	42	0.3	1	48	20.8	380.5	^{8}0.94025	0.4126
25	" 29, P.M.				9	2	3.5	[illegible]	7	[illegible]4.2	350.7	^{8}1.01275	0.4517
26	" " "	51.5°	48.25°		9	35	2.7	[illegible]	40	51.6	348.9	^{4}1.01160	0.4520
27	" " [illegible]				10	5	1.2	10	10	59.7	358.5	^{8}0.99495	0.4420
28	" " "				10	34	59.8	10	40	39.7	339.9	^{4}1.03800	0.4658
29	" " "				11	3	0.2	11	8	29.6	329.4	^{8}1.05505	0.4779
30	" " "				11	32	1.8	11	37	27.0	325.2	^{8}1.06920	0.4808
31	Nov. 11, P.M.	34°	41.25°		8	56	59.5	9	3	6.0	306.5	^{8}0.98370	0.4352
32	" " "				9	30	0.[illegible]	9	36	7.8	367.2	^{4}0.97820	0.4320
33	" " "				10	0	0.3	10	6	19.8	379.5	^{4}0.96700	0.4280
34	Nov. [illegible], P.M.			*Figure 4.* Width of the canal on the upstream side of the weir, 13.96 feet. Mean depth of the canal opposite the hook gauge boxes 5.048 feet below top of the weir. Two equal bays separated by a pier 1.02 feet wide.	9	12	1.3	9	19	37.8	456.5	^{5}1.01025	0.3527
35	" " "	45°	48°		9	59	59.6	10	7	18.4	438.8	^{5}1.02625	0.3500
36	Oct. 31, A.M.			*Figures 5, 6, and 7.* Width of the canal on the upstream side of the weir, 13.96 feet. Mean depth of the canal opposite the hook gauge boxes 2.014 feet below the top of the weir. Bottom of the canal horizontal for 28 feet on the upstream side of [illegible] weir.	7	17	15.8	7	22	52.9	337.1	^{3}1.02805	0.9496
37	" " "				7	47	59.6	7	53	29.9	330.3	^{3}1.03720	0.9589
38	" " "	46°	48.75°		9	16	0.8	9	51	31.0	334.3	^{8}1.04455	0.9684
39	" " "				10	14	1.4	10	19	23.3	321.9	^{3}1.04495	0.9698
40	" " "				10	41	0.7	10	46	28.2	327.5	^{3}1.04600	0.9691
41	" " "				11	10	7.8	11	15	32.5	324.7	^{5}1.05180	0.9756
42	" " "				11	39	59.4	11	45	8.3	308.9	^{3}1.07945	1.0049
43	" " P.M.				0	15	1.4	0	20	15.7	314.3	^{2}1.07115	0.9958

XIII.

LOWER LOCKS, LOWELL, IN OCTOBER AND NOVEMBER, [redacted]

Number of the experiment	9 Head due to the velocity of approach, or the values of h by the formula $h = \frac{v^2}{64.3236}$ Feet	10 Depth upon the weir, corrected for the velocity of the water approaching it, or the values of H' by the formula $H' = [(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}]^{\frac{2}{3}}$ Feet	11 Length of the weir l Feet	12 Number of end contractions n	13 Total quantity of water that passed the weir during each experiment, as measured in the lock chamber. Cubic feet	14 Quantity of water passing the weir per second Cubic feet per second	15 Value of C in the formula $Q = C(l - 0.1nH')H'^{\frac{3}{2}}$, Q having the corresponding values in column 14	16 Mean value of C for each position in description of weir	17 Approximate mean depth upon the weir, for each part in description of weir H'' Feet	18 Quantity of water that would have passed the weir with the depth in column 17, calculated by the formula $Q = C(l - 0.1nH'')H''^{\frac{3}{2}}$, C having the corresponding value in column 16 Cubic feet per second	19 Quantity of water passing the weir, calculated by the formula $Q = 3.33(l - 0.1nH'')H''^{\frac{3}{2}}$ Cubic feet per second	20 Proportional difference or the absolute difference of the quantities in columns 18 and 19, divided by the quantity in column 18
1	0.00917	1.53300	9.997	2	11815.19	61.2821	3.3318	3.3181	1.56	62.6147	62.8392	+0.0036
2	0.00910	1.55946	"	"	12069.49	62.5086	3.3174					
3	0.00960	1.56845	"	"	11964.90	63.2060	3.3230					
4	0.00968	1.57828	"	"	11512.52	63.3508	3.3002					
5	0.00512	1.24208	9.997	2	11723.21	45.0893	3.3412	3.3338	1.25	45.4125	45.3608	−0.0011
6	0.00517	1.24718	"	"	11758.00	45.3197	3.3398					
7	0.00531	1.25325	"	"	11725.57	45.6781	3.3405					
8	0.00519	1.25610	"	"	11760.23	45.4911	3.3159					
9	0.00560	1.25825	"	"	11958.73	45.9813	3.3396					
10	0.00557	1.26022	"	"	11908.41	45.8523	3.3260					
11	0.00282	0.96983	9.997	2	11872.75	31.1457	3.3265	3.3228	1.00	32.5486	32.6240	+0.0023
12	0.00328	1.03071	"	"	11615.35	33.9110	3.3129					
13	0.00339	1.03710	"	"	11869.83	34.2866	3.3110					
14	0.00331	1.03680	"	"	11715.20	34.2725	3.3182					
15	0.00341	1.04388	"	"	11713.35	34.6549	3.3196					
16	0.00339	1.04061	"	"	11651.45	34.5890	3.3283					
17	0.00279	0.96593	"	"	12023.62	30.9568	3.3241					
18	0.00288	[illegible]	"	"	11627.09	31.5377	3.3281					
19	0.00290	0.98229	"	"	11659.01	31.6579	3.3179					
20	0.00298	0.99172	"	"	11661.78	32.1438	3.3216					
21	0.00303	0.99752	"	"	11751.36	32.4706	3.3265					
22	0.00243	0.91801	"	"	11721.38	28.6789	3.3218					
23	0.00251	0.93012	"	"	11682.09	29.1929	3.3155					
24	0.00265	0.94681	"	"	11629.93	30.0004	3.3198					
25	0.00317	1.01581	"	"	11678.[illegible]0	33.3009	3.3211					
26	0.00318	1.01466	"	"	11323.48	33.3147	3.3281					
27	0.00305	0.99780	"	"	11670.08	32.5512	3.3338					
28	0.00337	1.03881	"	"	11637.13	34.4136	3.3207					
29	0.00355	1.05306	"	"	11685.17	35.1711	3.3269					
30	0.00368	1.07274	"	"	11764.18	36.1752	3.3283					
31	0.00294	0.98653	"	"	11702.07	31.9298	3.3251					
32	0.00290	0.98099	"	"	11620.66	31.6712	3.3259					
33	0.00279	0.96969	"	"	11762.21	30.9940	3.3111					
34	0.00193	1.01212	7.997	4	11803.64	25.9888	3.3617	3.3601	1.02	26.2686	26.0333	−0.0090
35	0.00201	1.02820	"	"	11655.85	26.5630	3.3586					
36	0.01402	1.04098	9.997	2	11747.38	34.8481	3.3519	3.3527	1.06	35.8026	35.5602	−0.0068
37	0.01430	1.05039	"	"	11657.37	35.2933	3.3498					
38	0.01458	1.05799	"	"	11963.56	35.7249	3.3548					
39	0.01461	1.05842	"	"	11513.09	35.7660	3.3567					
40	0.01460	1.05946	"	"	11715.10	35.7718	3.3523					
41	0.01480	1.06491	"	"	11712.43	36.0716	3.3548					
42	0.01570	1.09390	"	"	11579.32	37.4873	3.3509					
43	0.01512	1.08535	"	"	11615.21	37.0518	3.3505					

TABLE

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS, MADE AT THE

1	2	3		4	5						6	7	8
Number of the experiment.	Date of the experiment. 1852.	Temperatures by Fahrenheit's thermometer. Of the air in the shade.	Of the water.	Reference to the figures on plate XIV., and particular description of the weir.	Times of the commencement and conclusion of the experiment, as indicated by the [illegible]. Commencement. h.	min.	sec.	Conclusion. h.	min.	sec.	Duration of the experiment. sec.	Mean depth upon the weir by observation. H	Mean velocity of the water approaching the weir [illegible] in the hook gauge boxes, [illegible] feet from the weir. v. Feet.
44	Nov. 7, A.M.	41°	44°	*Figures 8, 9, and 10.* Mean width of the canal for 20 feet on the upstream side of the weir, 9.992 feet. Mean depth of the canal opposite the hook gauge boxes, 5.048 feet below the top of the weir.	7	50	1.0	7	55	51.2	350.2	^{8}0.98675	0.5155
45	“ “ “				9	38	0.0	9	43	59.7	359.7	^{4}0.98100	0.5110
46	“ “ “				10	11	59.2	10	17	57.4	358.2	^{8}0.97150	0.5370
47	“ “ “				10	43	59.6	10	49	59.0	359.4	^{8}0.97020	0.5385
48	“ “ “				11	15	0.0	11	21	0.4	360.4	^{8}0.97000	0.5387
49	“ “ “				11	48	4.4	11	54	0.7	356.3	^{4}0.97775	0.5391
50	“ “ P.M.				0	21	0.2	0	26	58.8	358.6	^{8}0.97000	0.5390
51	Nov. 7, P.M.	42.25°	43.75°	*Figures 8, 9, and 10.* Width and depth the same as the preceding. The sheet of water after passing the weir was prevented from expanding in width, for a certain distance, by boards at each end of the weir, placed in the same planes as the sides of the canal leading to the weir.	8	23	7.6	8	28	52.0	344.4	^{4}1.00507	0.5589
52	“ “ “				8	55	59.7	9	1	46.9	347.2	^{4}1.00600	0.5581
53	“ “ “				9	28	3.1	9	33	51.3	348.2	^{4}1.00520	0.5574
54	“ “ “				9	59	59.8	10	5	52.9	353.1	^{4}0.99205	0.5580
55	“ “ “				10	31	1.5	10	36	54.0	352.5	^{4}0.99210	0.5577
56	Oct. 24, P.M.			*Figures 1, 2, and 3.* Width of the canal on the upstream side of the weir, 13.96 feet. Mean depth of the canal opposite the hook gauge boxes, 5.[illegible] feet below the top of the weir.	2	21	59.3	2	29	53.8	474.5	^{4}0.81800	0.3405
57	“ “ “				3	3	0.4	3	11	11.1	490.7	^{4}0.80755	0.3348
58	“ “ “				3	40	0.8	3	48	10.0	[illegible]	^{4}[illegible]	0.3272
59	“ “ “				4	17	4.7	4	25	44.9	520.2	^{8}0.77030	0.3170
60	“ “ “				4	55	1.7	5	3	18.0	496.3	^{8}0.80125	0.3306
61	“ “ “				5	29	1.8	5	37	27.0	505.2	^{8}0.79100	0.3258
62	Oct. 31, P.M.			*Figures 5, 6, and 7.* Width of the canal on the upstream side of the weir, 13.96 feet. Mean depth of the canal opposite the hook gauge boxes, 2.014 feet below the top of the weir. Bottom of the canal horizontal for 28 feet on the upstream side of the weir.	2	20	0.3	2	28	40.5	520.2	0.77115	0.6691
63	“ “ “				3	0	1.3	3	8	32.4	511.1	^{8}0.78725	0.6872
64	“ “ “	47°	48.75°		3	38	4.4	3	46	7.9	483.5	^{8}0.80155	0.7052
65	“ “ “				4	14	0.2	4	21	5.5	425.3	^{8}0.87900	0.7870
66	“ “ “				4	47	58.0	4	54	56.2	418.2	^{8}0.88865	0.7908
67	Nov. 7, P.M.			*Figures 8, 9, and 10.* Mean width of the canal for 20 feet on the upstream side of the weir, 9.992 feet. Mean depth of the canal opposite the hook gauge boxes, 5.018 feet below the top of the weir.	2	7	2.7	2	16	14.7	552.0	^{8}0.73620	0.3859
68	“ “ “				2	18	1.0	2	26	6.8	485.8	^{4}0.80105	0.4122
69	“ “ “				3	17	51.7	3	25	50.0	478.3	^{2}0.80950	0.4173
70	“ “ “				3	51	59.9	3	59	51.7	471.8	^{4}0.81465	0.4213
71	“ “ “				4	25	0.0	4	32	[illegible]	[illegible]	^{9}0.81325	0.4102
72	Oct. 24, A.M.			*Figures 1, 2, and 3.* Width of the canal on the upstream side of the weir, 13.96 feet. Mean depth of the canal opposite the hook gauge boxes 5.[illegible] feet below the top of the weir.	7	12	2.4	7	25	0.4	778.0	^{8}0.59190	0.2182
73	“ “ “	46.5°	47.75°		7	49	59.8	8	2	52.7	772.9	^{8}0.59240	0.2183
74	“ “ “				9	11	59.1	9	24	36.6	757.5	^{8}0.61060	0.2273
75	“ “ “				10	33	59.7	10	45	14.4	674.7	^{8}0.65525	0.2503
76	“ “ “	59.5°	48°		11	8	1.4	11	19	27.9	686.5	^{4}0.64105	0.2449
77	“ “ “				11	50	0.8	12	1	35.9	695.0	^{4}0.63795	0.2419
78	“ “ P.M.	61.5°	48.5°		0	24	58.5	0	36	42.9	704.4	^{8}0.63870	0.2396
79	Oct. 31, P.M.			*Figures 5, 6, and 7.* Width of the canal on the upstream side of the weir 13.96 feet. Mean depth of the canal opposite the hook gauge boxes 2.014 feet below the top of the weir. Bottom of the canal horizontal for [illegible] feet on the upstream side of the weir.	7	6	59.6	7	18	8.6	669.0	^{8}0.65150	0.5405
80	“ “ “				7	46	0.3	7	57	9.3	669.0	^{8}0.65590	0.5455
81	“ “ “	45.25°	48.75°		8	24	0.0	8	34	56.9	656.9	^{8}0.65985	0.5490
82	“ “ “				9	0	59.4	9	12	42.4	703.0	^{4}0.63135	0.5198
83	“ “ “				9	40	1.0	9	51	27.8	686.8	^{8}0.64250	0.5309
84	“ “ “				10	23	1.7	10	34	11.0	669.3	^{8}0.65460	0.5439
85	Oct. 31, P.M.			*Figures 5, 6, and 7.* Width, depth, and bottom of the canal same as the preceding. Two equal bays separated by a partition [illegible] feet wide.	11	43	0.7	11	56	42.8	822.1	^{8}0.66910	0.4382
86	Nov. 1, A.M.				0	21	59.9	0	35	26.0	806.1	^{8}0.67900	0.4459
87	“ “ “				1	0	3.8	1	13	17.9	794.1	^{8}0.68300	0.4496
88	“ “ “				1	39	59.8	1	53	9.5	789.7	^{8}0.68815	0.4526

XIII — CONTINUED.

LOWER LOCKS, LOWELL, IN OCTOBER AND NOVEMBER, 1852.

		10	11	12	13	14	15	16	17	18	19	20
Number of the experiment	Head due to the velocity in column 8, or the values of *h* in the formula $h=\frac{v^2}{64.323}$	Depth upon the weir corrected for the velocity of the water approaching the weir, or the values of *H'* by the formula $H'=((H+h)^{\frac{3}{2}}-h^{\frac{3}{2}})^{\frac{2}{3}}$	Length of the weir *l*	No. of end contractions *n*	Total quantity of water that passed the weir during the experiment, as measured in the lock chamber	Quantity of water passing the weir per second	Value of *C* in the formula $Q=C(l-0.1nH')H'^{\frac{3}{2}}$, *Q* having the corresponding values in column 14	Mean value of *C* for each particular description of weir	Approximate mean depth upon each particular description of weir *H'*	Quantity of water that would have passed the weir with the depth in column 17, calculated by the formula $Q=C(l-0.1nH')H'^{\frac{3}{2}}$, *C* having the corresponding value in column 16	Quantity of water passing the weir, calculated by the formula $Q=3.33(l-0.1nH')H'^{\frac{3}{2}}$	Proportional difference, or the absolute difference of the quantities in columns 18 and 19 divided by the quantity in column 18
	Feet.	Feet.	Feet.		Cubic feet.	Cubic feet.			Feet.	Cubic feet per second.	Cubic feet per second.	
44	0.00468	0.99117	9.995	0	11524.62	32.9087	3.3366	3.3309	0.98	32.3950	32.2899	−0.0033
45	0.00161	0.98930	"	"	11616.54	32.8423	3.3391					
46	0.00119	0.97870	"	"	11592.18	32.8623	3.3437					
47	0.00151	0.98051	"	"	[illegible]635.28	32.[illegible]	3.3[illegible]18					
48*	0.00151	0.98081	"	"	[illegible]1689.70	32.1956	3.3134					
49	0.00152	0.98207	"	"	11576.77	32.1916	3.3102					
50	0.00152	0.98122	"	"	11623.39	32.4600	3.3413					
51	0.00186	1.00968	9.995	0	11[illegible]6.88	33.8179	3.3349	3.3270	1.00	33.2534	33.2833	+0.0009
52	0.00184	1.01001	"	"	11725.28	33.7708	3.3257					
53	0.00183	1.00980	"	"	11743.85	33.7279	3.3254					
54	0.00[illegible]67	0.99710	"	"	11688.18	33.0883	3.3219					
55	0.00[illegible]6	0.99681	"	"	11630.76	33.0[illegible]88	3.3243					
56	0.00180	0.82014	9.997	2	11539.45	24.3102	3.3287	3.3246	0.80	23.4011	23.4391	+0.0016
57	0.00178	0.80923	"	"	11675.86	23.7943	3.3234					
58	0.00166	0.79720	"	"	11628.76	23.2761	3.3237					
59	0.00150	0.77812	"	"	11[illegible]31.31	22.4801	3.3261					
60	0.00170	0.80450	"	"	11698.98	23.5427	3.3208					
61	0.001[illegible]	[illegible]500	"	"	11719.[illegible]	24.1700	3.3188					
62	0.00697	0.77768	9.997	2	11718.53	22.5270	3.3376	3.3403	0.83	24.8318	24.7548	−0.0031
63	0.00734	0.79412	"	"	11887.02	23.2577	3.3406					
64	0.00773	0.81178	"	"	11610.17	24.0128	3.3383					
65	0.00[illegible]63	0.88855	"	"	11695.06	27.4981	3.3435					
66	0.00380	0.89782	"	"	11671.63	27.9002	3.3417					
67	0.00268	0.74821	9.995	0	11676.57	21.1582	3.3368	[illegible]3308	0.80	23.3821	23.3156	−0.0028
68	0.00261	0.80449	"	"	11709.76	24.1041	3.3422					
69	0.00271	0.81211	"	"	11615.16	24.4492	3.3424					
70	0.00276	0.81700	"	"	11617.78	24.6870	3.3410					
71	0.00278	0.81588	"	"	11698.23	24.5581	3.3311					
72	0.00074	0.59262	9.997	2	11803.57	15.0001	3.3284	3.3275	0.62	16.0382	16.0502	+0.0008
73	0.00074	0.59812	"	"	11614.0[illegible]	15.0273	3.3308					
74	0.00081	0.61139	"	"	11902.18	15.7124	3.3281					
75	0.00[illegible]8	0.65021	"	"	[illegible]700.98	17.[illegible]805	3.3297					
76	0.00093	0.64895	"	"	11659.42	16.9839	3.3306					
77	0.00091	0.63888	"	"	11648.02	16.7597	3.3259					
78	0.00089	0.63456	"	"	11685.54	16.5894	3.3250					
79	0.00154	0.65579	9.997	2	11657.19	17.4248	3.3258	3.3262	0.65	17.1990	17.2187	+0.0011
80	0.00463	0.66027	"	"	11783.30	17.6133	3.3278					
81	0.00170	0.66[illegible]29	"	"	11674.77	17.7725	3.3278					
82	0.00419	0.63532	"	"	11682.49	16.6181	3.3249					
83	0.00138	0.6[illegible]664	"	"	11715.28	17.0578	3.3244					
84	0.00460	0.65894	"	"	11748.86	17.5540	3.3266					
85	0.00299	0.67226	7.997	4	11690.02	14.2197	3.3382	3.3368	0.68	14.4541	14.4217	−0.0020
86	0.00309	0.68195	"	"	11703.92	14.5192	3.3378					
87	0.00314	0.68060	"	"	11641.82	14.6642	3.3378					
88	0.00318	0.69118	"	"	11678.11	14.7880	3.3333					

COMPARISON OF THE PROPOSED FORMULA WITH THE RESULTS OBTAINED BY PREVIOUS EXPERIMENTERS

160. We find on record a great number of experiments on the discharge of water over weirs; in the present state of the science of hydraulics, however, a large proportion of them can be considered only in the light of first approximations; of great value undoubtedly, at the respective epochs at which they were made; but it could serve no useful purpose to compare them with the results obtained with the more perfect apparatus used of late years. Three sets of experiments have been made in France within the last thirty years, on a comparatively minute scale, it must be admitted, but with complete apparatus, and conducted with great care. They were made by Poncelet and Lesbros at Metz, in 1827 and 1828; by Castel at Toulouse, in 1835; and by Boileau at Metz, in 1846. It will be recollected that the application of the proposed formula to the discharge over weirs in which the contraction at the ends is complete, is limited to depths on the weir, not exceeding one third of the length of the sheet; this limitation permits the comparison to be made with only a portion of the results obtained by Poncelot and Lesbros, and by Castel. Boileau operated on weirs in which the end contraction was suppressed, and to which form the limitation does not apply.

161. *Comparison of the proposed formula, with the results obtained by Poncelet and Lesbros.* These experiments are to be found among the magnificent series made at the expense of the French Government, and recorded at length in *Es*[illegible] *hydrauliques sur les lois de l'ecoulement de l'eau* by M. M. Poncelet and Lesbros, Paris; 1832; and in the continuation under the same title by M. Lesbros, Paris; 1851. In table XXXIX., of the last mentioned work, are given the coefficients for computing the discharge over weirs of a variety of forms, and of certain lengths, and with certain depths of water, by the formula

$$d = m l h \sqrt{2gh},$$

in which d is the discharge, [illegible] the coefficient, l the length, h the depth, and $g = 9.8088$ metres, or 32.1817 feet. The comparison can be usefully made with only one of the forms experimented upon, namely: that in which the orifice was made in a thin plate, in the plane side of a reservoir; the orifice being at a great distance from the bottom and lateral sides, and the discharge made freely into the air.

In table XIV. are given the quantities computed according to Lesbros, for all the depths for which he gives values of m, determined by experiment, and which are within the limitation required by the proposed formula, namely: that the depth shall not exceed one third of the length. The quantities are also given [illegible] computed by the proposed formula. It will be perceived by the final column of the table, that the proportional differences are nearly constant, and that the quantities by the proposed formula are too small by a little more than two per cent. If the coefficient of the proposed formula was changed from 3.33 to 3.41, the computed results would agree very nearly. It should be recollected that the constants in the proposed formula have been determined from experiments in which the depths upon the weir were from 0.6 to 1.6 feet, or about eight times the depths in the experiments by Poncelet and Lesbros. It is the general result of all the precise experiments on the discharge through openings of a variety of forms, in a thin plate, that, for very small heads, the coefficients require to be increased; which proves that the law of the discharge varying as the square root of the head, does not hold good for very small heads. The comparison in table XIV. affords the same indications; and the constancy of the proportional differences, indicates that the correction of the length, to compensate for the effect of the end contraction, is practically correct, both for large and small depths upon the weir. It would not be difficult so to determine the values of the constants in the formula

$$Q = C(l - bnh)h^n,$$

[illegible] represent the experiments both of Poncelet and Lesbros and the Lower Locks experiments with nearly the same degree of exactness that the latter are represented, with the constants that have been adopted. This would undoubtedly be an advantage in some particular cases in practice, but if it was intended to make the formula general, the sacrifice of simplicity would be more than an equivalent disadvantage.

TABLE XIV.

The length of the weir is constant, and equal to 0.06562 feet.

1	2	3	4	5
Depth on the weir. Feet.	Value of the coefficient m according to Lesbros.	Quantity of water discharged by the formula $Q = m\,lh\sqrt{2gh}$, m having the corresponding value in the second column. Cubic feet per second.	Quantity of water discharged by the formula $Q = 3.33(L - 0.1nH)H^{\frac{3}{2}}$. Cubic feet per second.	Proportional difference, or [illegible]
0.06562	0.417	0.0569	0.0557	—0.0215
0.08202	0.414	0.0722	0.0706	—0.0225
0.09843	0.412	0.0870	0.0851	—0.0228
0.11483	0.409	0.0838	0.0820	—0.0207
0.13124	0.407	0.1019	0.0997	—0.0209
0.14764	0.405	0.1210	0.1184	—0.0212
0.16404	0.404	0.1419	0.1379	—0.0280
0.18045	0.402	0.1622	0.1583	—0.0243
0.19685	0.401	0.1844	0.1794	—0.0271
0.21326	0.399	0.2069	0.2012	—0.0274

162. *Comparison of the proposed formula with the results obtained by Castel.* An abstract of these experiments may be found in the *Annales de Chimie et de Physique*, vol. 62. Paris: 1836; and in the *Annales des ponts et chaussées*, vol. 1, for 1837. Paris. It appears to have been a leading idea in these experiments, to imitate, as nearly as possible, the forms and proportions of the weirs ordinarily used in practice for gauging streams of water; in fact, to reproduce them on a small scale, anticipating that the rules deduced from precise experiments upon them might be applied, without modification, to gaugings on a large scale. The weir was formed by damming up a wooden canal, 2.4279 feet in width, by a thin plate of copper, in which the weir was formed, the crest being 0.5578 feet above the bottom of the canal; the width of the weir varying from about ¼ of a foot to 2¼ feet. The latter width is so near that of the canal, that the end contraction must have been sensibly modified, so that any comparison of the results obtained from it would be of little use; they have consequently been omitted. In the abstract referred to, a table is given of the coefficients deduced from the experiments, for a variety of widths and depths. In table XV. are given the quantities computed with these coefficients, for all the widths and depths to which the proposed formula is applicable; also the quantities as computed by the proposed formula. In consequence of the small dimensions of the canal, the water approaching the weir had a sensible velocity; in table XV.

the depths on the weir, for which the quantities have been computed by the proposed formula, have been corrected for this velocity. It will be seen by referring to the final column, that the proportional differences are considerably greater, and have less uniformity than in the comparison with the experiments of Poncelet and Lesbros; nevertheless, there is a certain harmony in the results of both comparisons, and they serve to show how unsafe it is, in the present state of the science of hydraulics, to apply rules to gauging streams of water passing over weirs, of which the dimensions differ greatly from those in the experiments from which the rules have been deduced.

17

TABLE XV.

Width of the canal leading to the weir 2.4279 feet; height of the crest of the weir above the bottom of the canal 0.5578 feet.

1	2	3	4	5	6	7	8
Length of the weir Feet	Depth on the weir Feet	Values of the coefficient n in the formula $Q=n\frac{2}{3}LH\sqrt{2gH}$, according to Castel	Quantity of water discharged by the formula $Q=n\frac{2}{3}LH\sqrt{2gH}$, n having the corresponding values given in the preceding column Cubic feet per second	Height due to the velocity of the water in the canal leading to the weir [illegible] $h=\frac{v^2}{64.38}$ Feet	Depth on the weir corrected for the velocity of the water [illegible] by the formula $H'=((H+h)^{\frac{3}{2}}-h^{\frac{3}{2}})^{\frac{2}{3}}$ Feet	Quantity of water discharged by the formula $Q=3.33(L-0.1nH')H'^{\frac{3}{2}}$ Cubic feet per second	Proportional differences [illegible] column 7 divided by the quantities in column 4
0.3281	0.09843	0.618	0.0335	0.00001	0.09844	0.0317	—0.0537
0.6562	0.19685	0.604	0.1852	0.00016	0.19701	0.1796	—0.0302
“	0.16404	0.611	0.1425	0.00010	0.16414	0.1380	—0.0316
“	0.13124	0.619	0.1033	0.00006	0.13130	0.0998	—0.0338
	0.09843	0.621	0.0673	0.00003	0.09846	0.0655	—0.0268
0.9843	0.32809	0.604	0.5976	0.00120	0.32924	0.5778	—0.0331
“	0.26247	0.606	0.4290	0.00072	0.26319	0.4189	—0.0237
“	0.19685	0.610	0.2805	0.00036	0.19720	0.2755	—0.0178
“	0.16404	0.616	0.2155	0.00023	0.16426	0.2109	—0.0213
“	0.13124	0.623	0.1559	0.00014	0.13138	0.1519	—0.0257
“	0.09843	0.631	0.1026	0.00006	0.09849	0.0993	—0.0322
1.3124	0.39371	0.621	1.0769	0.00317	0.39687	1.0266	—0.0468
“	0.32809	0.621	0.8192	0.00225	0.33022	0.7876	—0.0386
“	0.26247	0.620	0.5852	0.00131	0.26375	0.5682	—0.0290
“	0.19685	0.622	0.3813	0.00067	0.19749	0.3720	—0.0245
“	0.16404	0.626	0.2920	0.00044	0.16446	0.2842	—0.0266
“	0.13124	0.632	0.2109	0.00025	0.13148	0.2042	[illegible]
“	0.09843	0.636	0.1379	0.00012	0.09855	0.1332	—0.0341
1.6404	0.32809	0.613	1.0105	0.00369	0.33177	1.0016	—0.0088
“	0.26247	0.632	0.7457	0.00218	0.26452	0.7192	—0.0355
“	0.19685	0.632	0.4843	0.00108	0.19788	0.4692	—0.0312
“	0.16404	0.633	0.3690	0.00069	0.16470	0.3578	—0.0304
“	0.13124	0.636	0.2653	0.00037	0.13161	0.2566	—0.0327
“	0.09843	0.642	0.1740	0.00019	0.09861	0.1671	—0.0397
1.9685	0.32809	0.644	1.2748	0.00545	0.33308	1.2174	—0.0450
“	0.26247	0.644	0.9118	0.00326	0.26513	0.8725	—0.0431
“	0.19685	0.645	0.5931	0.00163	0.19838	0.5675	—0.0432
“	0.16404	0.644	0.4505	0.00103	0.16502	0.4320	—0.0410
“	0.13124	0.645	0.3229	0.00058	0.13179	0.3094	—0.0417
“	0.09843	0.651	0.2117	0.00027	0.09860	0.2012	—0.0495

163. *Comparison of the proposed formula, with that obtained by Boileau.* The experiments from which Boileau deduced his formula, are given at length in *Jaugeage des cours d'eau a faible ou à moyenne section,* by M. P. Boileau. Paris: 1850. Boileau has particularly studied the discharge in the form of weir in which the contraction at the ends is suppressed; that is to say, the form in which the weir occupies the whole width of the canal conducting the water to it. The proposed formula is applicable to this case, by making $n = 0$. Boileau experimented on three weirs of this form; one of them was 5.30 feet in length, with the crest 1.54 feet above the bottom of the canal; the other two were 2.94 feet in length, the crest in one being 1.12 feet above the bottom of the canal; and in the other 1.61 feet above the bottom, the depths on the weir varying from 0.10 foot to 0.72 feet. By a train of reasoning combined with the results of his experiments, Boileau has arrived at the following formula for weirs of this form:—

$$Q = \frac{S - H}{\sqrt{(S+H)^2 - H^2}} \, 0.417 \, L H \sqrt{2 g H},$$

in which

Q = the discharge.
S = the height of the crest of the weir, above the bottom of the canal, which is supposed to be horizontal for a short distance, upstream from the weir.
H = the depth on the weir, taken before the sheet begins to curve in consequence of the discharge.
[illegible] = the width of the canal, and also the length of the weir.
$g = 9.8088^m$.

The coefficient 0.417 is determined from a mean of 14 experiments.

Adopting the English foot as the unit, and reducing, we have

$$Q = 3.3455 \frac{S - H}{\sqrt{(S+H)^2 - H^2}} L H^{\frac{3}{2}}. \qquad (A)$$

For this form of weir, the proposed formula becomes

$$Q = 3.33 \, L H'^{\frac{3}{2}}: \qquad (B)$$

H' being the depth upon the weir, corrected for the velocity of the water approaching the weir.

These formulas differ so essentially that they can be conveniently compared only by applying them to particular cases. In the formula (*A*), as S increases relatively to H, the factor $\frac{S-H}{\sqrt{(S+H)^2-H^2}}$ approaches unity, which is the limit when S is infinitely greater than H; in the latter case we have also, $H'=H$; the formulas (*A*) and (*B*) then become identical, excepting the coefficients, that in (*B*) being $\frac{1}{216}$ less than in (*A*). Hence we may conclude that for any length of weir, and for any depth upon it, providing that the depth of the canal leading to the weir, is very great relatively to the depth on the weir, the quantities computed by the formulas (*A*) and (*B*) will differ $\frac{1}{216}$ only.

In practice, however, S is seldom very great, relative to H. Let us take an example conforming more nearly to the usual cases that occur in practice. Let $H=1$ foot, $S=3$ feet, $L=10$ feet, by the formula (*A*), $Q=34.552$ cubic feet per second. In the formula (*B*), H' is the depth on the weir, corrected for the mean velocity of the water approaching the weir; this velocity is equal to the quotient of the area of the section of the canal, divided by the quantity. But the quantity itself depends on this velocity. The formula (*B*), if put under a form to give the quantity directly from the measured depth upon the weir, would become very complicated; it will be equally exact and much easier, to find the quantity by successive approximations as follows.

1st approximation.

Assume $H'=1$, then $Q=33.3$.

2nd approximation.

If $Q=33.3$, the mean velocity of the water in the canal leading to the weir is $\frac{33.3}{10(H+S)}=0.8325$; and for the head due this velocity we have

$$h=\frac{(0.8325)^2}{2g}=0.011;$$

$$H'=\left[(H+h)^{\frac{3}{2}}-h^{\frac{3}{2}}\right]^{\frac{2}{3}}=1.0103;$$

$$Q=33.816.$$

A third approximation in a similar manner gives $Q=33.817$.

The proportional difference of the quantities by the two formulas is about $\frac{1}{47}$, or a little over two per cent.

Boileau, in establishing his formula, assumes that the living force in the entire section of the canal is expended in increasing the discharge over the weir; in the method adopted in this work for correcting the depth on the weir for the velocity of the water in the canal, it is assumed that the living force in the part of the section of the canal equal to the area of the orifice of discharge only, is expended in increasing the discharge; as applied to a weir of the form under consideration, it is clear that neither of these assumptions is strictly true; the latter, however, appears to be the most rational, and to agree the best with experiment.

PRECAUTIONS TO BE OBSERVED IN THE APPLICATION OF THE PROPOSED FORMULA.

104. $$Q = 3.33\,(L - 0.1\,n\,H)\,H^{\frac{3}{2}}:$$

in which

Q = the discharge, in cubic feet per second;
L = the length of the weir;
n = the number of end contraction;
H = the depth on the weir;

the English foot being the unit of measure.

When the contraction is complete at each end of the weir, $n = 2$; when the weir is of the same width as the canal conducting water to it, the end contraction is suppressed, and $n = 0$.

[illegible] formula is only applicable to rectangular weirs, made in the side of a dam, which is vertical on the upstream side, the crest of the weir being horizontal, and the ends vertical; also, the edges of the orifice presented to the current must be sharp; for, if bevelled or rounded off in any perceptible degree, a material effect will be produced on the discharge; it is essential, moreover, that the stream should touch the orifice only at these edges, after passing which it should be discharged through the air, in the same manner as if the orifice was cut in a thin plate.

The formula is not applicable to cases in which the depth on the weir exceeds one third of the length; nor to very small depths. In the experiments from which it has been determined, the depths have varied from 7 inches to nearly 19 inches, and there seems no reason why it should not be applied with safety to any depths between 6 inches and 24 inches.

The height of the surface of the water in the canal, above the crest of the weir, is to be taken for the depth upon the weir; this height should be taken at a point far enough from the weir to be unaffected by the curvature caused by the discharge; if more convenient, it may be taken by means of a pipe opening near the bottom of the canal near the upstream side of the weir, which pipe may be made to communicate with a box placed in any convenient situation; and if the box and pipe do not leak, the height may be observed, in this manner, very correctly (art 175). However the depth may be observed, it may require to be corrected for the velocity of the water approaching the weir.

The end contraction must either be complete, or entirely suppressed; the necessary distance from the side of the canal or reservoir to the end of the weir, in order that the end contraction may be complete, is not definitely determined; in experiments 1 to 4, table XIII., the depth on the weir was about 1.5 feet, and the distance from the side of the canal to the end of the weir, about 2 feet; the proposed formula applies well to all these experiments. In cases where there is end contraction, we may assume a distance from the side of the canal to the end of the weir equal to the depth on the weir, as the least admissible, in order that the proposed formula may apply.

As to the fall below the weir, requisite to give a free discharge to the water, it is not definitely determined; a comparison of experiments 49, 50, and 51, table X., indicates that, when the depth on the weir is 1 foot, and the entire sheet, after passing the weir, strikes a solid body at about 0.5 feet below the crest of the weir, the discharge, with the same depth, is diminished about $\frac{1}{100}$. By experiments 1 and 2, table XII, it appears that, when the sheet passing the weir, falls into water of considerable depth, the depth on the weir being about 0.85 feet, no difference is perceptible in the discharge, whether the water is 1.06 feet or 0.235 feet below the crest of the weir; it is very essential, however, in all cases, that the air under the sheet should have free communication with the external atmosphere. With this precaution it appears that, if the fall below the crest of the weir is not less than half the depth upon the weir, the discharge over the weir will not be perceptibly obstructed. If the sheet is of very great length, however, more fall will be necessary, unless some special arrangement is made to supply air to the space under the sheet at the places that would otherwise not have a free communication with the atmosphere.

In respect to the depth of the canal leading to the weir, experiments 36 to 48, table XIII., show that, with a depth as small as three times that on the weir, the proposed formula agrees with experiment, within less than one per cent.; this proportion may be taken as the least admissible, when an accurate gauging is required.

It not unfrequently happens that, in consequence of the particular form of the canal leading to the weir, or from other causes, the velocity of the water in the canal is not uniform in all parts of the section; this is a frequent cause of serious error, and is often entirely overlooked. If great irregularities exist, they should be removed by causing the water to pass through one or more gratings, presenting numerous small apertures equally distributed, or otherwise, as the case may require, through which the water may pass under a small head; these gratings should be placed as far from the weir as practicable.

If the canal leading to the weir has a suitable depth, it will be requisite only when great precision is required, to correct the depth upon the weir for the velocity of the water in the canal by the formula (*D*) (art. 153); thus, in experiment 42, table XIII, the water in the canal had a mean velocity of about 1 foot per second, the effect of which was to increase the discharge about two per cent.; in experiment 82, in which the velocity was about 0.5 feet per second, the discharge was increased about one per cent; these examples will enable the operator to judge, in each case, of the necessity of going through the troublesome calculation for correcting the depth on the weir.

MISCELLANEOUS EXPERIMENTS ON THE FLOW OF WATER, MADE AT THE LOWER LOCKS, IN NOVEMBER, 1852.

On the discharge of water over a dam of the same section as that erected by the Essex Company, [redacted] the Merrimack River at Lawrence, Massachusetts.

165. As these experiments cannot be usefully compared with those on weirs of more regular form, they have not been included in table XIII.; and as they are of less general interest, they will not be given with much detail.

The form of the dam is represented by figures 11 and 12, plate XIV. (art. 147); the other apparatus was the same as that used for the experiments in table XIII.

The end contraction was suppressed by making the canal leading to the overfall of the same width as the overfall itself. The water in the hook gauge boxes communicated only with the water contained in the spaces between the masonry and the wood-work forming the sides and bottom of the canal leading to the overfall; as there was a free communication between the water at *A*, figures 11 and 12, and that near the hook gauge boxes, and as the water between these places was sensibly at rest, we may consider that the height of the water was taken at *A*

166. In table XVI. these experiments are exhibited in sufficient detail to be intelligible.

Columns 1 and 2 require no explanation

Column 3. The heights contained in this column are above the mean level of the crest of the dam, which was very nearly horizontal for a distance of 2.95 feet from *C* to *D*. These heights have not been corrected for the velocity of the water approaching the weir; indeed, from the manner in which they were observed, no correction was necessary.

Column 4. The quantities in this column have been obtained in the manner described in the explanation of table XIII. (art. 155).

Column 5. *Quantity of water passing over the dam, calculated by the formula*

$$Q = 3.01208\, l h^{1.53}.$$

This formula was arrived at by trial of various powers of h, and was adopted as representing, the most nearly, the results of the five experiments in the table; it should be distinctly understood, however, that it is not applicable to depths much greater or less than in the experiments from which it is deduced. In April, 1852, the depth of water flowing over the dam at Lawrence, was 10 feet; if the quantity then passing over the dam were computed by this formula, it is probable that it would be greatly in error.

COLUMN 6. *Proportional difference.* It will be observed that the greatest proportional difference is 0.0085, or less than one per cent.; we may therefore say with confidence, that we can compute the flow of water over the Lawrence dam, when free from ice or other obstruction, for any depth not greater than 20 inches or less than 7 inches, without being liable to an error exceeding one per cent.

TABLE XVI.

Time, from November 10th, 8h, 57', P.M., to November 11th, 6h, 11', A.M.
Temperature of the air at 10h, 50' P.M., 34.50° Fahrenheit.
" water " 41.76°
The air calm.

1	2	3	4	5	6
Number of the experiment.	Length of the overfall. l.	Mean height of the surface of the water in the hook gauge boxes, above the top of the horizontal crest of the dam. Feet. h.	Quantity of water passing over the dam as measured in the lock chamber. In cubic feet per second.	Quantity of water passing over the dam calculated by the formula $Q = 3.01208\, l\, h^{1.53}$. In cubic feet per second.	Proportional difference, or the absolute difference of the quantities in columns 4 and 5 divided by the quantity in column 4.
89	9.995	0.58720	13.385	13.332	−0.0040
90	"	0.79035	20.892	21.005	+0.0054
91	"	0.97670	28.914	29.039	+0.0043
92	"	1.32520	46.189	46.317	+0.0028
93	"	1.63380	64.346	63.804	−0.0085

EXPERIMENTS TO ASCERTAIN THE EFFECT OF TAKING THE DEPTHS UPON A WEIR AT DIFFERENT DISTANCES FROM IT, BY MEANS OF PIPES OPENING NEAR THE BOTTOM OF THE CANAL.

167. It is often a matter of great doubt and uncertainty, to know at what distance from the weir the depth of the water upon it should be observed; very often also it becomes a matter of necessity to observe the depth at a distance from the weir so small that, according to some, the quantity of water passing the weir, computed in the usual manner, would be liable to sensible

error. For the purpose of obtaining some light upon this point these experiments were undertaken, and, as they were made with all the precautions for insuring accuracy that could be devised, they will be described with some detail.

168. Figures 8, 9, and 10, plate XIV., represent the form of the weir, and the system of pipes used for these experiments. The canal leading to the weir was of the same width as the weir, so that the end contraction was suppressed. The pipes were of lead, about three fourths of an inch interior diameter, the lower extremities of which, numbered from 1 to 8, were about three inches above the bottom of the canal, and terminated in holes in the board *CC*; the side of the board at which they opened was vertical, and in the axis of the canal; the ends of the pipe did not project through the board; the other extremities of the pipes were fastened by small flanges to the bottoms of the hook gauge boxes; holes were made in the bottoms of the boxes corresponding to each pipe, and communication between the boxes and the pipes could be controlled at pleasure, by plugging up these holes. It will be readily perceived that heights of the water observed by this apparatus are not necessarily the true elevations of the surface of the water immediately over the orifices of the pipes, but that they are the elevations of the surface in the hook gauge boxes; an elevation which is due to the statical pressure on the orifice of the pipe.

169. In order to obtain the heights at different distances from the weir, observations were necessarily made with both hook gauges at the same time, one of which was always in communication with a pipe opening at 6 feet from the weir, the apertures in the bottom of the box, communicating with all the other pipes, being plugged up; at the other hook gauge, either pipe might be in communication with the box, all the other apertures being plugged up; the depth at six feet from the weir was observed in each experiment, to be used as a standard with which the depth observed simultaneously at any other distance might be compared; this mode of proceeding was rendered necessary, in consequence of the impossibility of maintaining the level of the water uniform for any considerable length of time.

170. In considering the sources of error to which the observations with the hook gauges were liable, it appeared that four kinds required to be specially guarded against, namely: *First*, imperfect comparison of the gauges, with the top of the weir. *Second*, defective stability, in consequence of which the relative elevations of the gauges and the weir might not be constant. *Third*, errors in the graduation of the gauges. *Fourth*, the difference in the habit of observers, in making the point of the hook coincide with the surface of the water; or, what we may call, the personal error. In relation to the *first*, we must bear in mind

that the requirement here is not so much that the absolute height above the top of the weir should be exactly determined, [illegible] that the difference of the heights at two points, at different distances from the weir, should be determined correctly, if then we know the relative heights of the two gauges, the object can be attained, even if we do not know precisely the height of either of them, relatively to the weir. The heights of the gauges relative to each other, could easily be ascertained at any time, by closing up all the apertures in each box, except those communicating with pipes, numbers 4 and 5, which, it will be seen by reference to figure 9, had a common orifice at their lower extremities; consequently, the surface of the water in both boxes must have been at the same level. The correction to be applied to the reading of one of the hook gauges, was taken as previously determined for the experiments on the discharge over the weir, and the correction for the other gauge, was deduced from simultaneous observations on both gauges, when the boxes communicated with [illegible] common orifice, in the manner just described. The *second* source of error was guarded against as much as practicable, by making the observations for the correction just described, at nearly the same time [illegible] the experiments to which it was to be applied. The danger of error from the *third* source was much diminished by making the observations for the correction, with nearly the same depth upon the weir as in the experiments to which it was to be applied. The *fourth* source of error was eliminated by determining the correction separately for each pair of observers. In short, these four sources of error were reduced to a minimum by determining for each session of the experiments, and for each pair of observers, the relative corrections to be applied to the readings of the hook gauges, to the depths upon the weir; the depths, when the observations for these corrections were made, being nearly the same [illegible] in the experiments to which they were to be applied.

171. In table XVII. are given the results of the observations made for the purpose of obtaining the relative corrections for the gauges, for each session of the experiments, and for each pair of observers In computing the depth upon the weir by the north hook gauge, the correction -0.03072 is applied to the mean reading of the gauge, (art. 148); the mean reading of the south hook gauge is given; as the water in both boxes is at the same height; the difference between the depth upon the weir, as determined by the north hook gauge, and the mean reading of the south hook gauge, must give the correction for the last named gauge.

TABLE XVII.

DATE 1852	Time of beginning the observation.	North hook gauge, in communication with pipe No. 3, opening near the bottom of the flume, and 6 feet from the weir.		South hook gauge, in communication with pipe No. 4, opening near the bottom of the flume, and 6 feet from the weir.			
		Observer	Arithmetical mean of depths on the weir. Feet	Observer	Arithmetical mean of readings of the gauge. Feet	Correction to be applied to the readings of the gauge, to give the depth on the weir. Feet.	Mean correction for each series and each pair of observers. Feet.
November 3. " "	9h 18′ P.M. 10 0 "	Francis "	1.01180 1.02617	Avery "	1.03760 1.05375	—0.02580 —0.02758	—0.02669
November 3.	11h 16′ P.M.	Haeffely	1.00739	Newell	1.03377	—0.02638	—0.02638
November 3. " " " 4.	9h 20′ P.M. 10 45 " 1 47 A.M.	Francis " "	1.01984 1.01073 1.04532	Newell " "	1.04625 1.03716 1.07169	—0.02641 —0.02648 —0.02637	—0.02640
November 3. " 4.	11h 0′ P.M. 1 58 A.M.	Francis "	1.00807 1.04734	Haeffely "	1.03431 1.07350	—0.02624 —0.02616	—0.02620
November 7. " " " " " " " " " " " " " "	7h 50′ A.M. 9 38 " [illegible] 7 P.M. 8 22 " 8 56 " 9 28 " 10 0 " 10 31 "	Francis " " " " " " "	0.98775 0.98555 0.73665 1.00696 1.00677 1.00580 0.99338 0.99294	Avery " " " " " " "	1.01362 1.01195 0.76157 1.03287 1.03311 1.03244 1.01973 1.01961	—0.02587 —0.02610 —0.02602 —0.02591 —0.02634 —0.02664 —0.02635 —0.02667	—0.02630
November 7. " " " "	8h 4′ A.M. 9 49 " [illegible] 26 P.M.	Francis " "	0.98932 0.98019 0.78315	Newell " "	1.01478 1.00597 0.80906	—0.02546 —0.02578 —0.02591	—0.02572
November 7.	9h 20′ A.M.	Haeffely	0.99305	Newell	1.01997	—0.02692	—0.02692

172. It will be perceived, by an examination of table XVII., that there are greater irregularities in the comparisons by some observers, than in those by others; this is to be attributed, principally, to the different degrees of experience and skill in the observers.

173. In table XVIII. are given the details of the experiments, to ascertain the effect of observing the depths upon the weir, at different distances from the weir, by means of pipes opening near the bottom of the canal. In order to obtain the depth upon the weir by the north hook gauge, the correction —0.03072 has been applied to the mean readings of this gauge. The correction for the south hook gauge is taken from the final column of table XVII., for the corresponding session and pair of observers. From want of time, pipes number 6 and 7 were not made use of.

It will be perceived, by referring to the final column of table XVIII., that the differences in the heights, at the different distances tried, are very inconsiderable, and such as could be detected only by the most delicate means of observation.

174. Two comparisons were made in a similar manner, of the heights, when one gauge box communicated with a pipe opening near the bottom of the canal, and the other with a pipe opening through the side, at about 4.2 feet above the bottom, the orifices of both being at [illegible] feet from the weir, as represented at *B*, figures 8, 9, and 10, plate XIV.; the following are the results.

First comparison, made November 7th, beginning at 3h, 52′, P.M.

Francis, at north hook gauge, with pipe No. 5, depth on weir	0 81616 feet
Avery, at south hook gauge, with pipe *B* “ “	0.81641 “
Difference	+ 0 00025 feet.

Second comparison, made November 7th, beginning at 4h, 5′, P.M.

[illegible]ncis, at north hook gauge, with pipe No. 5, depth on weir	0.81775 feet.
Nowell, at south hook gauge, with pipe *B* “ “	0.81776 “
Difference	+ 0.00001 feet.

These differences are so minute that we may conclude that the depth was the same whether the pipe opened near the bottom of the canal or at 4.2 feet above.

175. These experiments, taken in connection with those of Boileau,* who has arrived at similar results, leave no doubt as to the propriety, whenever convenience requires it, of observing the depths upon the weir by means of a pipe opening into the dead water, near the bottom of the canal on the upstream side of the weir.

---

* *Jaugeage des cours d'eau, by M P Boileau.* Paris: 1850.

TABLE XVIII.

DATE, 1852	Time of beginning the observation.	North hook gauge.			South hook gauge.			Difference in the corrected depths on the weir, [illegible] standard	Mean difference [illegible] being the standard
		[illegible]			[illegible]				
		Number of the pipe	Observer	Corrected depth on the weir.	Number of the pipe	Observer	Corrected depth on the weir.		
November 3	11h 53′ P.M.	5	Francis	1.01267	1	Newell	1.01321	+ 0.00054	
“ 4	0 27 A.M.	5	“	1.01439	1	Haeffely	1.01459	+ 0.00020	
“ 7	10 33 “	5	Haeffely	0.97530	1	Newell	0.97547	+ 0.00017	+ 0.00033
“ “	10 44 “	5	Francis	0.97644	1	Avery	0.97683	+ 0.00039	
“ “	10 56 “	[illegible]	“	0.97658	1	Newell	0.97695	+ 0.00037	
November 4	0h 37′ A.M.	5	Haeffely	1.02189	[illegible]	Newell	1.02286	+ 0.00097	
“ “	1 23 “	5	Francis	1.01220	[illegible]	Haeffely	1.01263	+ 0.00043	+ 0.00050
“ “	1 34 “	[illegible]	Haeffely	1.04472	2	Newell	1.04481	+ 0.00009	
November 7	11h 48′ A.M.	5	Francis	0.97829	3	Avery	0.97883	+ 0.00054	
“ “	0 21 P.M.	[illegible]	“	0.97734	3	[illegible]	0.97800	+ 0.00066	+ 0.00060
November 4	1h 5′ A.M.	[illegible]	Francis	1.03882	4	Newell	1.03940	— 0.00058	
“ “	1 11 “	8	Haeffely	1.03701	4	“	1.03881	— 0.00180	
“ 7	11 9 “	8	Francis	0.97501	4	“	0.97671	— 0.00170	— 0.00152
“ “	11 15 “	[illegible]	“	0.97579	4	Avery	0.97761	— 0.00182	
“ “	11 31 “	[illegible]	“	0.97530	4	Newell	0.97700	— 0.00170	
November 7	2h 48′ P.M.	5	Francis	0.80266	1	Avery	0.80281	+ 0.00015	
“ “	3 0 “	[illegible]	“	0.80731	1	Newell	0.80806	+ 0.00075	+ 0.00045
November 7	4h 25′ P.M.	[illegible]	Francis	0.81046	2	Avery	0.81132	+ 0.00086	
“ “	4 41 “	[illegible]	“	0.80972	2	Newell	0.81079	+ 0.00107	+ 0.00096
November 7	3h 18′ P.M.	[illegible]	Francis	0.80984	4	Avery	0.81072	— 0.00088	
“ [illegible]	3 36 “	[illegible]	“	0.81362	4	Newell	0.81491	— 0.00129	— 0.00108

A METHOD OF GAUGING THE FLOW OF WATER IN OPEN CANALS OF UNIFORM RECTANGULAR SECTION, AND OF SHORT LENGTH.

176. The motion of water in canals of uniform section, and of great length, has been successfully investigated by several eminent philosophers and engineers; all their calculations, however, are based on certain fundamental conditions, relating to the uniformity of the motion, either in different parts of the same section, or in the different sections;[*] conditions which are not generally fulfilled in canals of which the length is short in proportion to the other dimensions It is a matter of common observation, that the irregularities in the motion of water caused by changes in the form of the channel, or in the direction of the current, do not cease immediately after passing the cause of the irregularity. For instance, if the water at its entrance into a canal has, from any cause, a greater velocity on one side of the canal than on the other, the irregularity will disappear only at a certain distance from the entrance, depending upon the particular circumstances; this distance may be equal only to the width of the canal, or it may be at ten times that distance; and it is only after the water [illegible] traversed a sufficient distance to become free from such irregularities, that [illegible] usual rules relating to its motion are applicable.

177. The volume of water passing a given section of a canal, is equal to the product of the area of the section by the mean velocity, taken in a direction perpendicular to the plane of the section; in straight canals, of uniform section, and of great length, the motion of the water, after passing through a certain length of the canal, becomes regular and uniform, in which case the mean velocity may be deduced, with considerable accuracy, from the velocity of the surface at the centre of the canal, where the current is the most rapid. In short canals, however, this method is rarely applicable, if much precision is required. Various other methods have been devised for obtaining the mean

---

[*] See *A Treatise on Water-Works by Charles S. Storrow.* Boston. 1835. Page 24.

velocity in such cases,* affording approximations more or less perfect. It has been frequently found convenient at Lowell, to gauge large streams of water by causing them to flow through short rectangular canals of uniform section, and a particular method of obtaining the mean velocity has been practised, which will now be described.

178. A convenient part of the feeding canal is selected and lined with timbers and planks, so as to make a smooth and uniform rectangular channel; this is called a flume. The mean velocity is obtained by means of tubes, loaded at one end, so that they may float in nearly a perpendicular position, the lower ends just clearing the bottom of the flume; these tubes are put in near the upper end of the flume, and from the observed paths and velocities that they assume through a defined portion of the length of the flume, a mean velocity is deduced. Three of these flumes are represented at figure 1, plate XI. *I* is the flume used to gauge the water drawn by the Middlesex Company through the penstock *H*; *L* is the flume for gauging the water drawn by the Massachusetts Cotton-Mills; and *M* the flume for gauging the water drawn by the Boott Cotton-Mills; one side and the bottom of each of the two latter flumes, are permanently attached to the canal. Whenever a measurement is desired, the partition is put in; it is also continued down the canal about 900 feet beyond the lower end of the flume, or to a point just below the lower penstock, through which water is drawn by the Massachusetts Cotton-Mills; at which point it is connected with the bank. These two establishments are all that draw from this canal. When the partition is in place, and impervious to water, all the water that passes the flume *L*, is drawn by the Massachusetts Cotton-Mills; and all that passes the flume *M*, is drawn by the Boott Cotton-Mills. *N*, gratings for the purpose of equalizing, [illegible] certain extent, the velocity of the water entering the flumes; these are not necessary in all situations. The fall at the grating is usually from 2 to 4 inches, and the level of the water in the basin *A*, is raised an equal amount whenever a measurement is made. *O*, floats or rafts made of planks, for the purpose of diminishing the oscillations in the surface, caused by the gratings. *P* and *R*, timbers for the upstream and downstream transit stations; the upstream sides of these timbers form vertical planes, marking definite parts of the flumes, in which the mean velocities are to be measured; these timbers, and also the intermediate timbers *Q*, are graduated into feet, the zero points being in the axes of the

---

* See *Weisbach's Principles of the Mechanics of Machinery and Engineering.* Vol. 1, ch. VIII. Philadelphia: 1848.

respective flumes; this graduation is for the purpose of being able to observe the paths of the tubes. *S*, gauges, placed in boxes communicating with the flumes, for the purpose of showing the depth of water in the flumes. The times of the transits of the tubes at both of the stations *P* and *R*, were observed by the same chronometer, the signals being made by an electric telegraph erected for the purpose. The telegraph used for this purpose, is a very simple apparatus; the circuit is formed by an insulated copper wire, about $\frac{1}{80}$ of an inch in diameter, and the electric current is maintained by a small galvanic battery. Whenever the circuit is broken, a small electro magnet becomes demagnetized, which causes a slight blow to be struck on a vertical glass plate, placed near the observer, who notes the times of the transits; apart from the precision of the results obtained by this method of giving signals, it is quite an advantage to be able to station the observer at some quiet and convenient spot in the neighborhood.

The tubes are cylinders, made of tinned plate, about two inches in diameter, and of a length usually a little exceeding the depth of the water in the flume. The elevations of every part of the bottom being known, an inspection of the gauge *S* shows at once the height of the surface of the water in the flume above the mean level of the bottom, and also above the highest part of the bottom. The tubes are loaded with lead at one end, so as to sink in stagnant water to a depth about an inch less than the depth of the water on the highest part of the bottom. Whenever the level of the water is liable to vary, several sets of tubes are prepared of different lengths; generally, however, at Lowell, the level can be maintained at a uniform height with sufficient exactness, during the period required for measurement. The length of the tube is such as to project about four inches above the surface of the water; on some occasions they have accidentally been much longer. If the position assumed by the tubes varied much from the vertical, it would become sensible in such cases; it has seldom been perceived, however, evidently because the centre of gravity of the tube is so far below the centre of gravity of the space occupied by the immersed portion of the tube.

The operation of observing the passage of a tube is as follows. An assistant, standing upon the float *O*, or on a platform in about the same situation if no float is used, places the tube in as nearly a vertical position as he can, immerses it to the required depth, and at any required point in the width of the canal; a manœuvre requiring a little practice to perform it in a satisfactory manner. The tube being abandoned to the current, reaches the transit station *P*, where an observer, at the instant it passes, touches the key of the "break circuit" connected with the wire of the telegraph, which causes a signal to be made in the vicinity of the chronometer, at which the time of the transit is noted. Another assistant notes the distance from the axis of the flume, at which the tube passes the upper transit station. The tube passes on to the intermediate station *Q*, where an observer notes its distance from the axis. At the lower station *R*, the observations of the transits, and the distance from the axis of the flume, are taken in the same manner as at the upper station *P*. An assistant standing on a platform a little on the downstream side of the station *R*, catches the tubes as they leave the flume.

179. In deducing the mean velocity from the observations on the passage of the tubes, it is assumed that each tube moves with exactly the mean velocity of the water throughout the whole depth of the canal, in the particular path followed by the tube; the distance traversed by the tube being measured on a line parallel to the axis of the flume. An obvious objection to this assumption is, that the tube does not extend to the bottom, and that, consequently, a portion of the stream in which the water has the least velocity, has no direct effect upon the velocity of the tube, which will therefore assume a velocity somewhat above the mean for the whole depth. Weisbach, in his *Principles of the Mechanics of Machinery and Engineering*, previously cited, volume 1, chapter VIII, has the following remark, which is well founded, and has a direct relation to the subject under consideration. "As a rule, especially with large and floating bodies, as ships, etc., the velocity of the swimming body is somewhat greater than that of the water; not so much because these bodies in swimming float down an inclined plane formed by the surface of the water, but because they take none, or scarcely any, part in the irregular intimate motion of the water; still, the variation for small floating bodies is [illegible] slight that it may be neglected." Again, if the water has different velocities at different depths, it is plain that, at some parts of [illegible] length of the tube, the water will be passing by it, causing a pressure on the upstream side of the tube; and at other parts of its length, the tube will be moving through the water, causing a pressure on its downstream side; these two pressures will generally be in equilibrium The pressure on any point of the tube, will, according to well-known principles, be proportional to the square of the relative velocity of the tube and of the current, at the same point; consequently, if we suppose the depth to which the tube is immersed to be divided into a great number of equal laminæ, we shall have the sum of the squares of the relative velocities of all the laminæ that are moving faster than the tube, equal to the sum of the squares of the relative velocities of all the laminæ that are moving slower than the tube. The mean velocity of the current for the whole depth to which the tube is immersed, is equal to the arithmetical mean of the velocities of all the laminæ. The velocity assumed by the tube, depends upon

a different law; hence, it is evident that, generally, the tube will assume [illegible] velocity different from the mean velocity of the stream, for the depth to which the tube is immersed.

Although it is obvious that, whenever the velocities at different depths do not vary much, as is usually the case in canals, the difference between the velocity of the tube, and the mean velocity of the stream, for the depth to which the tube is immersed, must be small; still, there [illegible] a degree of uncertainty attending it. It is also unknown what correction to make, in consequence of the tube not reaching to the bottom of the canal. In order to estimate the degree of accuracy of this mode of finding the mean velocity, and also to ascertain what correction to apply to the results of such gaugings, experiments have been made at Lowell, on a scale of unusual magnitude. The mean velocity of the water passing a flume, has been obtained by means of the tubes, in the manner described above; and from this, and the known dimensions of the flume, the quantity of water flowing through it, has been deduced. After passing the flume, the same water was caused to flow over weirs of regular form, by which means an independent gauge of the quantity passing the flume, was obtained, by a method known to be sufficiently exact for most purposes. The apparatus with which these experiments were made, together with the method of conducting them, will now be described.

180. The mean velocity of the water passing the Middlesex flume, was obtained by means of the tubes, in the manner described above. In figure 1, plate XI., the flume is represented at *I*, [illegible] it was in the four experiments made August 15th, 1852. In these experiments the arrangements were the same, as for [illegible] gauging of the water drawn by the Middlesex Company; excepting that the head of the penstock *H* was carefully closed up, and the water passing the flume, conducted to the gauging weirs, which were the same in all the experiments. Figures 2 and 3, plate XI, are a plan and longitudinal section of the flume and gauging weirs, as they were during the seven experiments [illegible] September 5th, 1852, in which, the width of the flume was diminished to about 10 feet, by a partition placed nearly in the centre.

After passing the flume, the water was conducted through the upper lock chamber to the basin *K*, which had gauging weirs on three of its sides; the total length of these weirs was 77.884 feet. The crests of the weirs were not less than 5 feet above the highest parts of the bottom of the basin, the depths on which were observed by means of four hook gauges attached to the posts *L*, figures 2, 3, and 4; the depths on the weirs, in none of the experiments, much exceeded a foot. The guides *N*, figures 2 and 4, were for the purpose of facilitating the change of direction of the current entering the basin *K*, which, it will be perceived by reference to figure 2, had to turn at right angles to reach two of the weirs. The quantity of water gauged at these weirs, varied in the different experiments from 120 to 298 cubic feet per second. In gauging the larger quantities, it would have been more satisfactory if the basin *K* had been more extensive, as the change of direction in the current caused a sensible inequality in the heights of the water, in different parts of the basin; however, as the depths upon the weirs are deduced from observations made at the four gauges symmetrically placed in relation to the weirs, only a slight error can have arisen from this cause. Precautions were taken to prevent leakage between the flume and the measuring weirs, and the change in the quantity, from this cause, must have been entirely insensible.

Before commencing an experiment, the velocity in the flume was regulated, if necessary, by placing planks across the lock, at the hollow quoins *O*; no change whatever was made in these planks during an experiment. Figures 1, 2, and 3, show the form of the approaches and of the gratings, at the heads of the flumes. These arrangements had an important effect upon the relative velocities in different parts of the section of the flume, as will be explained further on In the experiments made August 15th, the distance between the upstream and downstream transit stations, was 110 feet; in the experiments of September 5th, this distance was reduced to 100 feet; the change being made in the position of the upstream transit station

181. With a view of rendering these experiments more intelligible, the details of the observations made at the flume, for the purpose of obtaining the mean velocity in experiment 5, are given at length in table XIX., which contains the greater part of the reductions of the observations necessary, in order to find the mean velocity.

Explanation of table XIX. The greater part of the columns are sufficiently explained by the respective headings.

Column 6. The mean distances given in this column, are found by adding twice the distance at the middle station to the distances at the two other stations, and dividing the sum by 4.

Column 8. For the purpose of simplifying the reduction, arithmetical means are taken of all the mean distances of the side tubes that fall between the even feet from the centre; and the same also, with the corresponding ratios of the velocities of the side tubes to the centre tubes. The quantities in column 8 are represented by the diagram, figure 4, plate XV., which is reduced from the original on a much larger scale. The mean distances are [illegible] for the abscissas; those on the north side of the centre, *minus;* those on the south side, *plus.* The ratios of the velocities are taken for the ordinates, the upper extremities of which are connected by the broken line; the full line connects the extremities of the ordinates, corrected, when necessary, for the most striking irregularities of the observations In tracing the corrected line, a weight is attributed to the ordinates, as they are given in column 8, proportional to the number of observations from which they are respectively deduced; thus, the ordinate deduced from all the observations with the centre tubes, which is taken as unity, is determined from twenty-four observations; while the adjacent ordinates on each side are determined from a mean of two observations each; consequently, it is assumed that the ordinate corresponding to the centre tubes, is determined with a much greater degree of certainty than the adjacent ordinates, and in drawing the full line for the corrected ordinates, a corresponding weight has been given to it The ordinates terminated by the full line, are assumed to represent the relative mean velocities of the water, at distances from the centre of the flume equal to their corresponding abscissas.

COLUMN 9. The numerical values of the ordinates of the full line in figure 4.

COLUMNS 10, 11, and 12, are for the purpose of finding the mean velocity of the whole section, that of the centre tubes being unity. This mean will evidently be the area of the figure $ABCDE$ divided by the sum of the positive and negative abscissas, or $AB = 9.98$. The sum of all the partial areas in column 12, or the area of the figure $ABCDE = 9.23647$, which, divided by 9.98, gives 0.9255 for the mean velocity of the whole section, that of the centre tubes being unity. The mean time occupied by the centre tubes in passing 100 feet, was 25.11 seconds; they had, therefore, a mean velocity of $\frac{100}{25.11} = 3.9825$ feet per second. The mean depth of the water in the flume during the experiment, was 8.007 feet; the mean section of the flume was, consequently, $9.98 \times 8.007 = 79.91$ square feet; and the quantity of water passing the flume,

$$79.91 \times 3.9825 \times 0.0255 = 294.5 \text{ cubic feet per second.}$$

182 During the whole period occupied in the observations at the flume, four observers were taking the depths upon the weirs, by means of the hook gauges; from these observations, and the known dimensions of the weirs, the mean quantity computed by the formula

$$Q = 3.33\,(L - 0.1\,n\,H)\,H^{\frac{3}{2}},$$

was found to be 286.44 cubic feet per second, or about 8 feet per second less than by the measurement in the flume; or in other words, the measurement in the flume, by means of the tubes, gave a result about 2¼ per cent greater than the measurement at the weirs.

The tubes used for determining the mean velocity in this experiment, were immersed 7.875 feet; consequently, there was a lamina of water, between the lower ends of the tubes and the bottom of the flume, of the mean thickness

$$8.007 - 7.875 = 0.132 \text{ foot}$$

The velocity of this lamina could have had no direct influence on the velocity of the tube; and as it is probable that its velocity was less than that of the mean of the laminæ above it, it is fair to presume that, if the tubes had been immersed to a sufficient depth to include this bottom lamina, it would have had a mean velocity a little slower than was found by experiment, and the agreement of the measurement at the flume, with that at the weir, would have been nearer; this, however, will account for only a part of the difference.

183 Table XIX. contains, also, observations with centre floats. These were of wax, about 2 inches in diameter; the mean time occupied by them in passing 100 feet, was 26.81 seconds; giving a mean surface velocity of

$$\frac{100}{26.81} = 3.73 \text{ feet per second,}$$

or a little over 6 per cent *slower* than the mean velocity as deduced from the observations with the tubes. Prony gives a formula for deducing the mean velocity of the whole section from the [illegible] locity; this formula, when reduced to the English foo[illegible] ie unit, becomes

$$[illegible] = \frac{V(V+7.78188)}{V+10[illegible]};$$

in which, $V =$ the surf[illegible], and $v =$ the mean velocity; substituting 3.73 for V, we find

$$v = 3.0507;$$

giving as the quantity passing the flume,

$$79.91 \times 3.0507 = 243.78 \text{ cubic feet per second,}$$

which is 15 per cent less than by the weir measurement. To such gross errors are we liable, if we misapply the rules of hydraulics.

---

* See *A Treatise on Water-Works, by Charles S. Storrow.* Page 96. Boston: 1835.

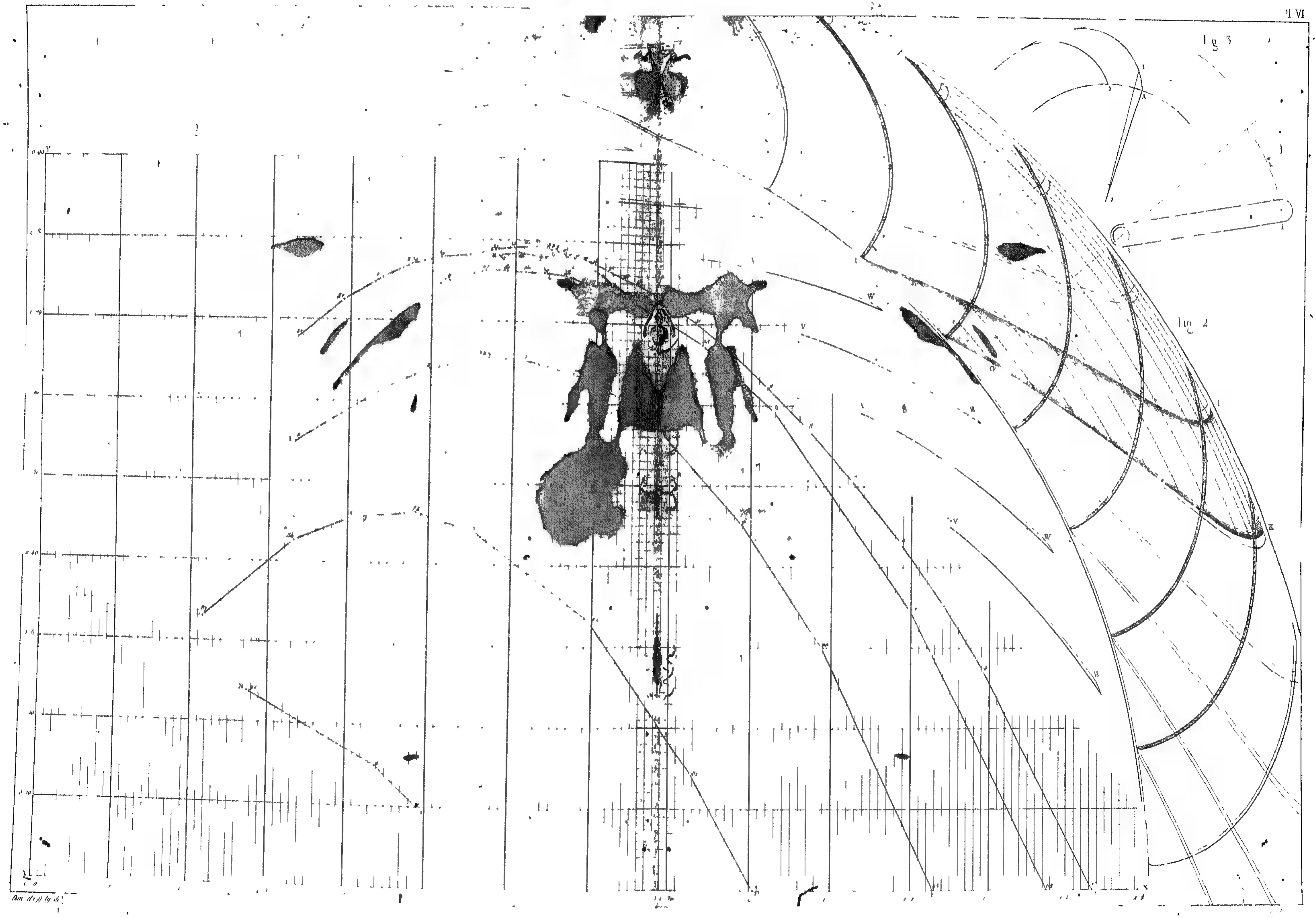
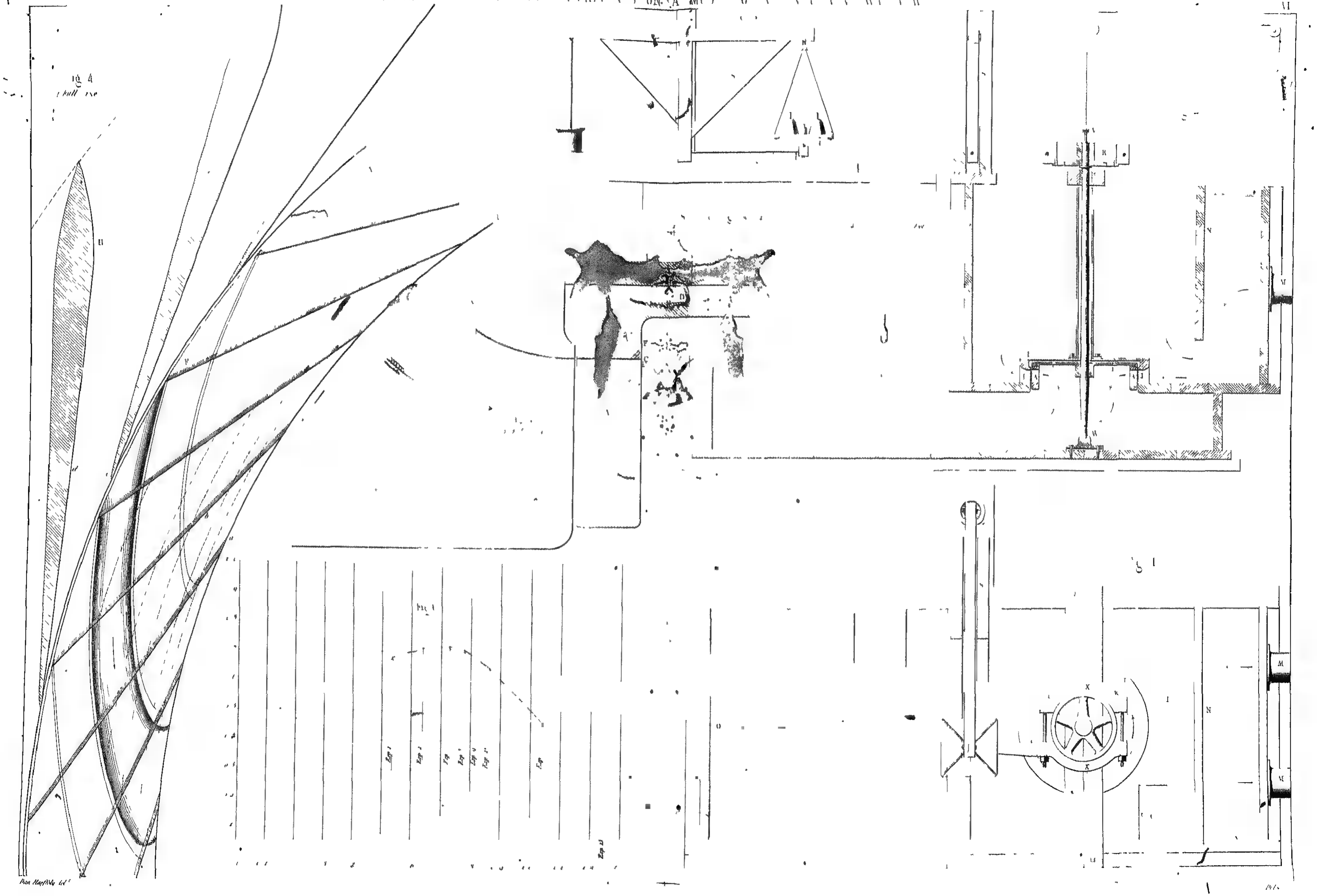


Fig 3
Fig 2
A
W
V
O
N
K

[illegible] ON A M[illegible]



Day & Haghe Lith.

CENTRE VENT WHE[illegible] COTTON MILLS

Pl. VIII.

Fig. 3

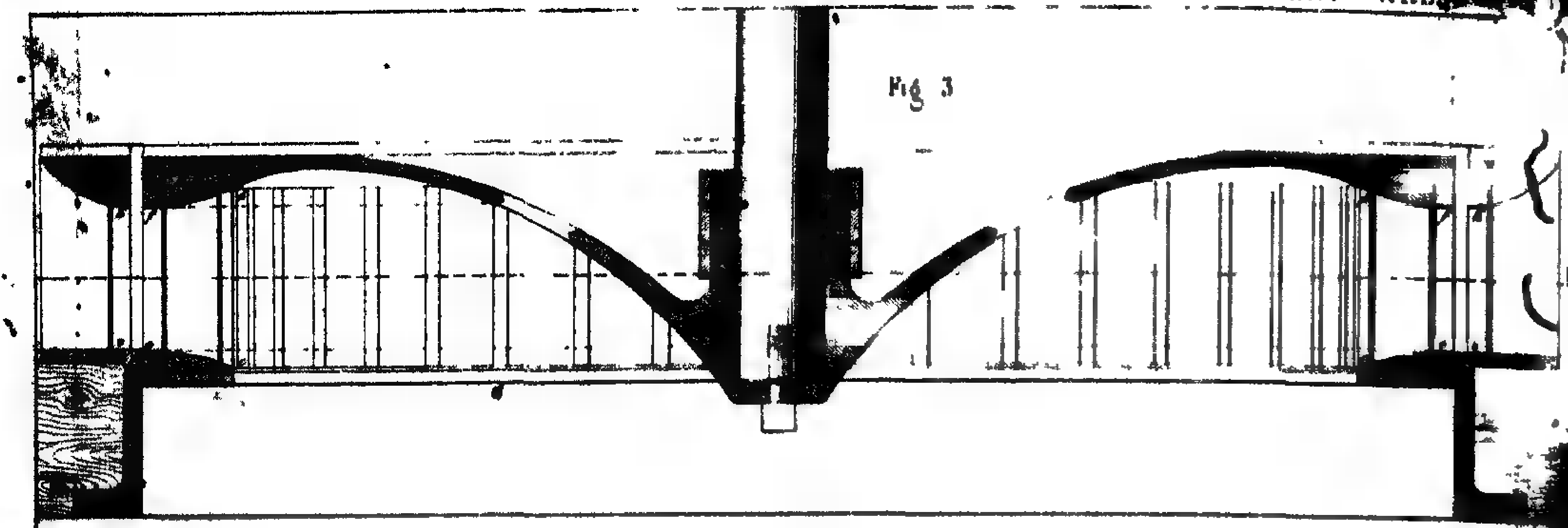
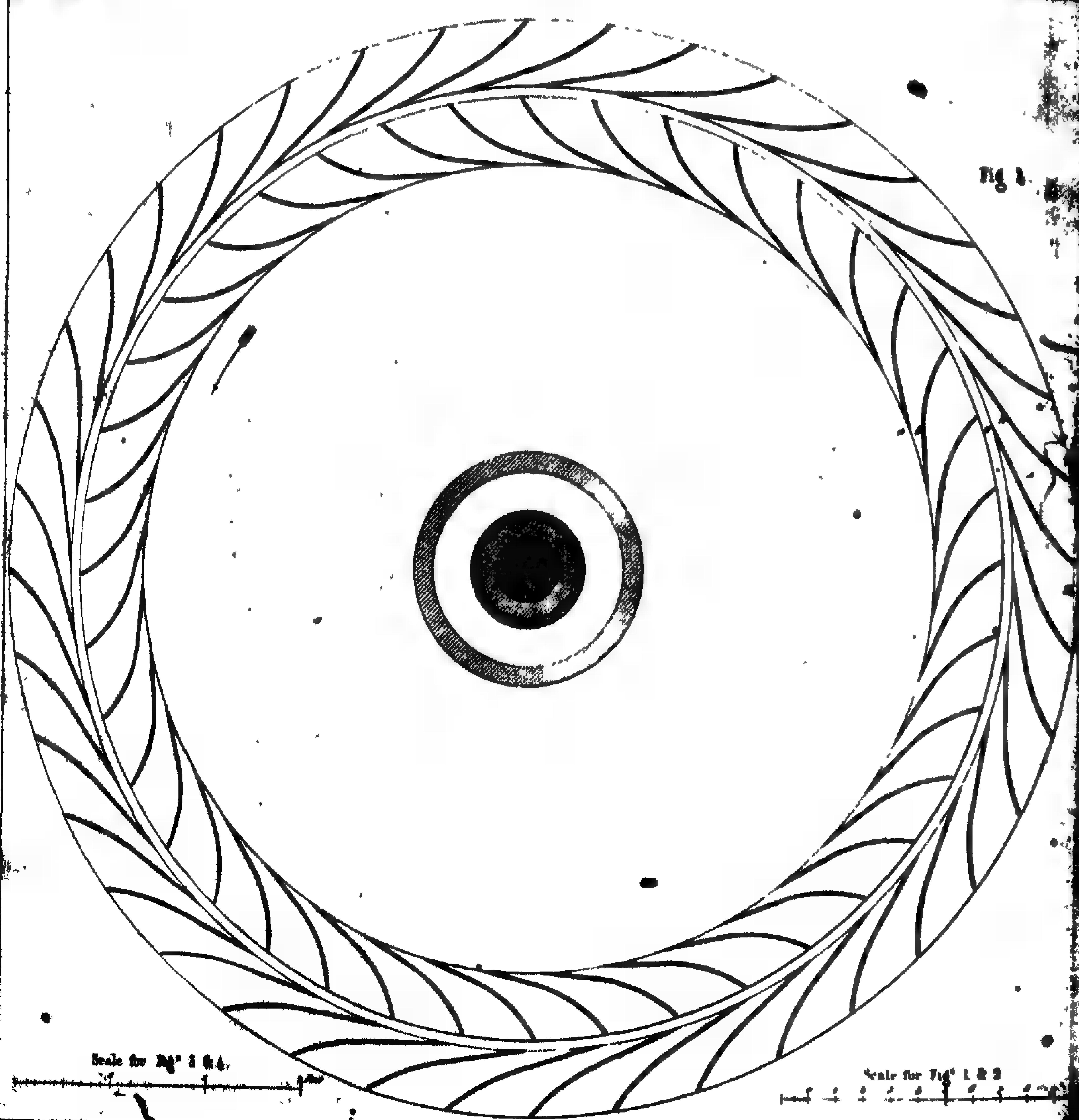


Fig. 4.



Scale for Figs 3 & 4.

Fig. 1.

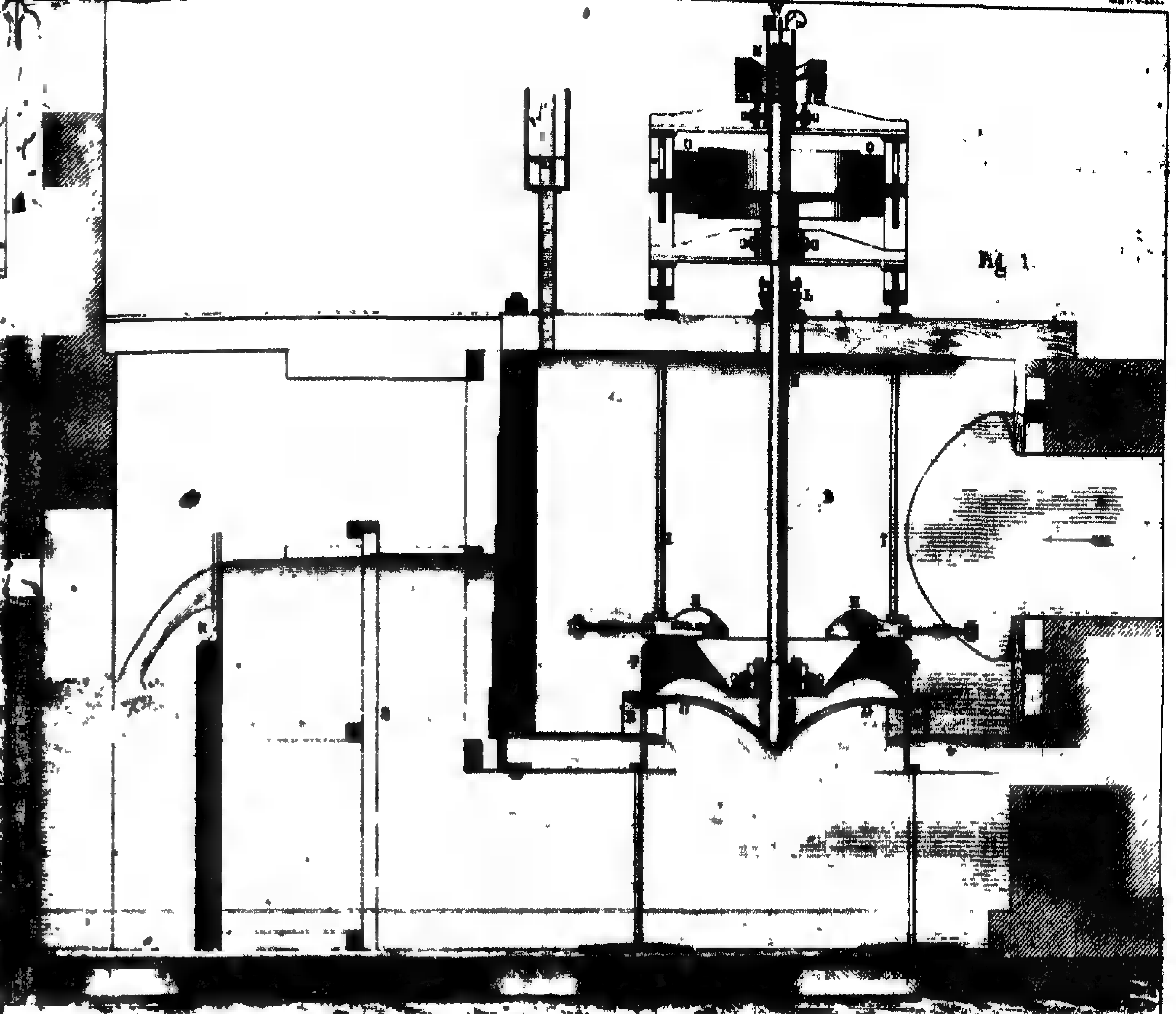
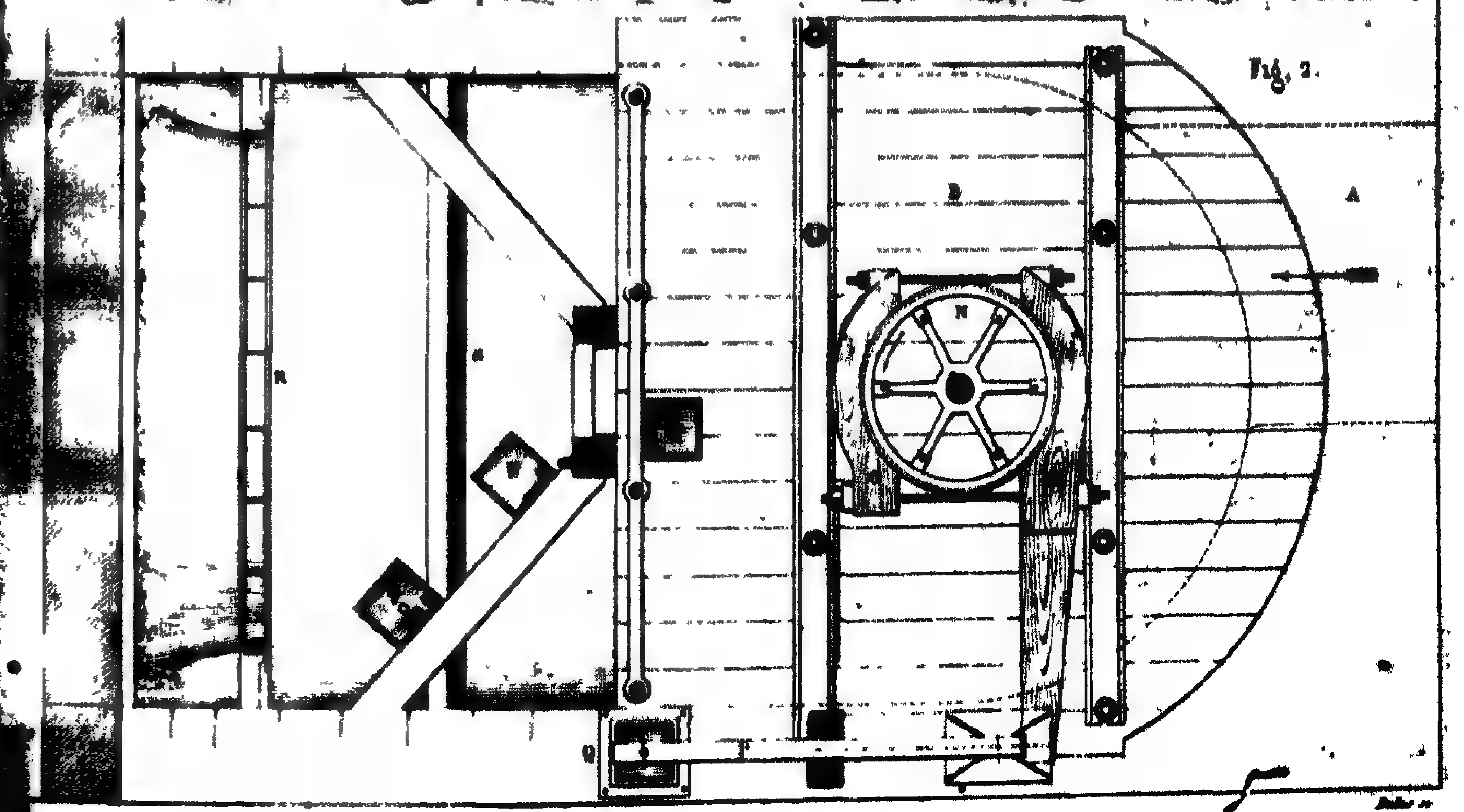


Fig. 2.



Scale for Figs 1 & 2

Dan. Harffely del.

EXPERIMENTS ON THE CENTRE V[illegible]ATER [illegible]EEL AT THE BOOTT COTTON MILLS

Pl. IX.

CHANGES IN THE FORM OF TH[illegible]IR AT THE TREMONT TURBINE

Fig 1

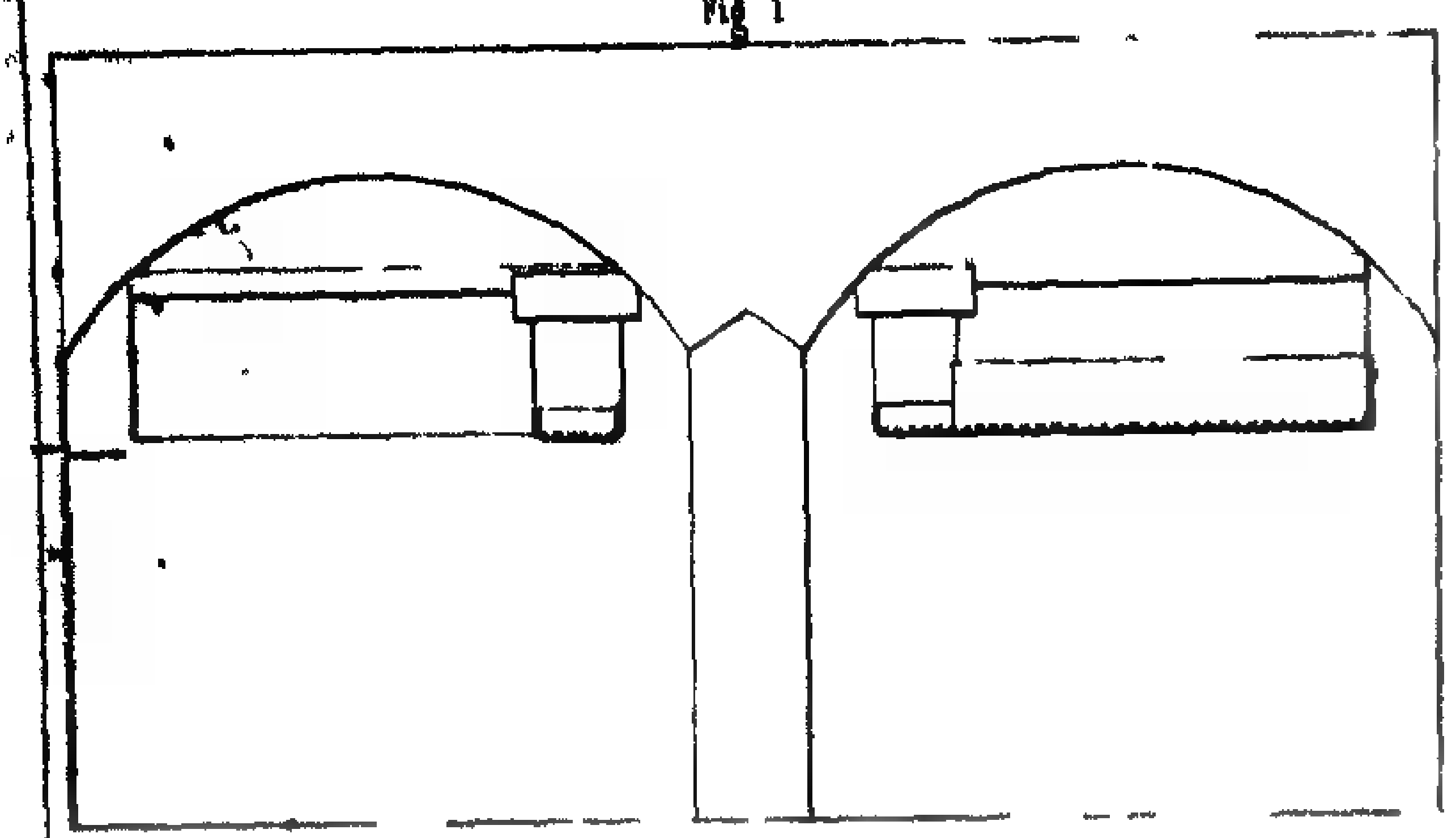


Fig 2

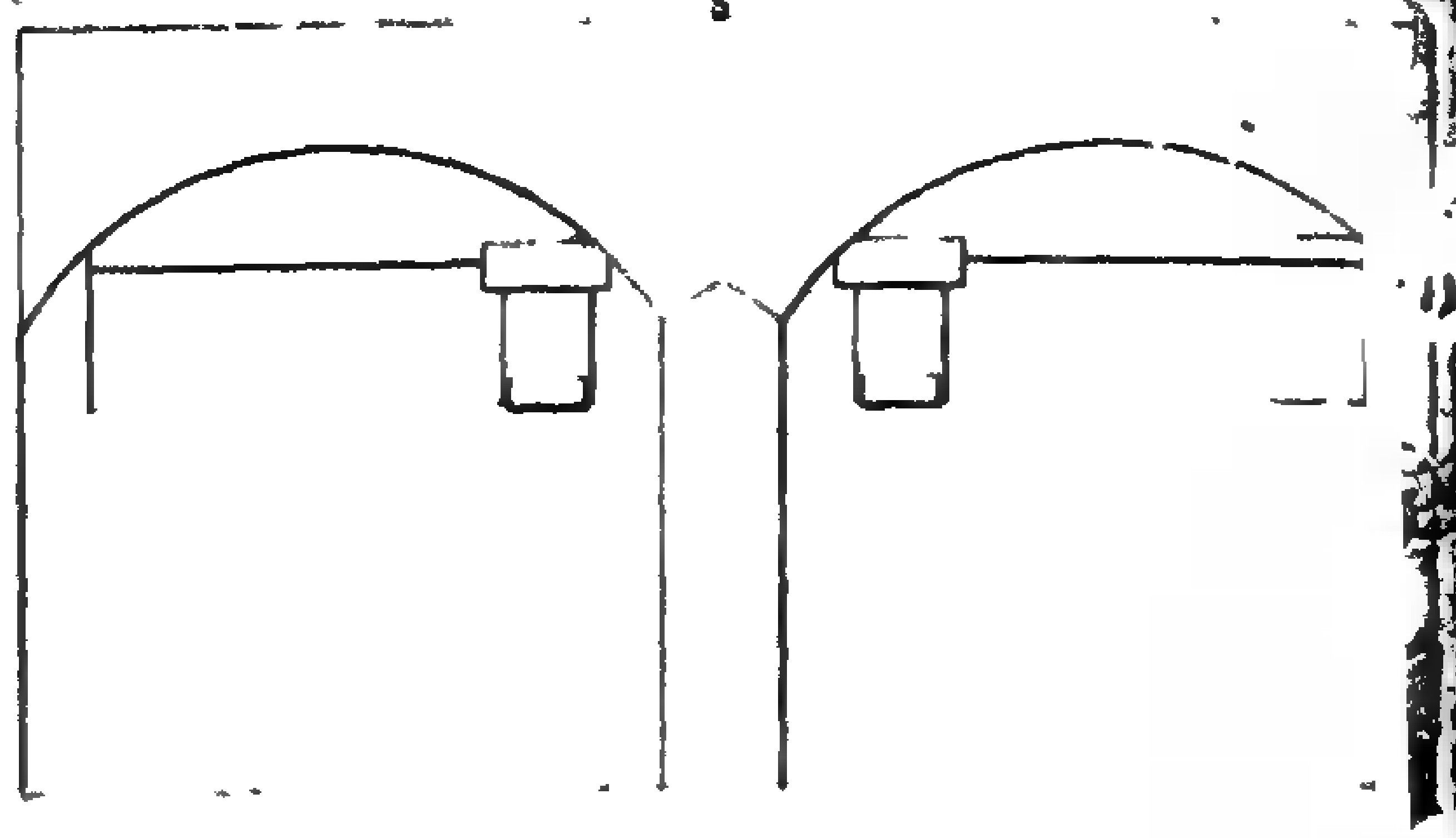


Fig 3

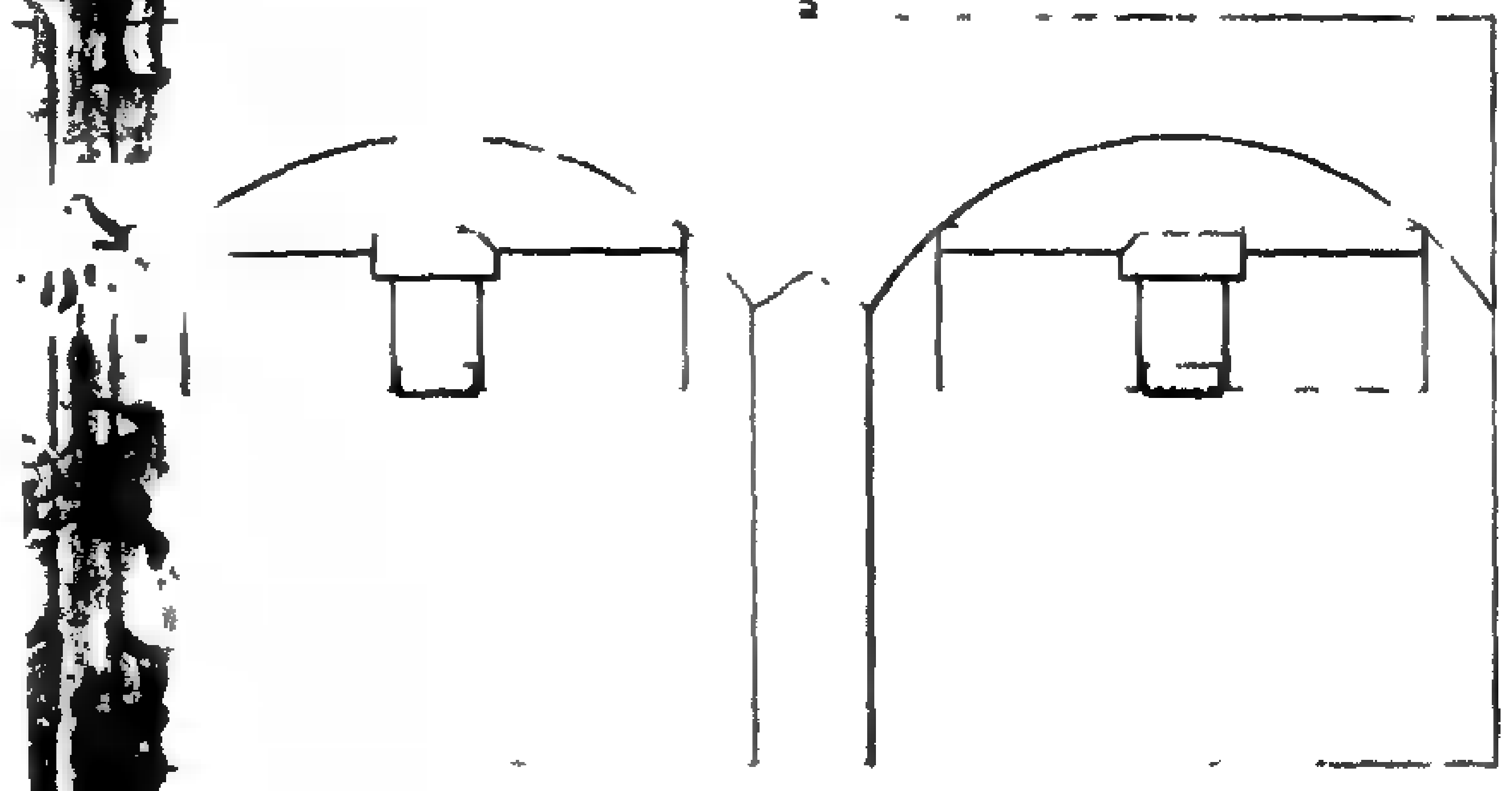


Fig 4

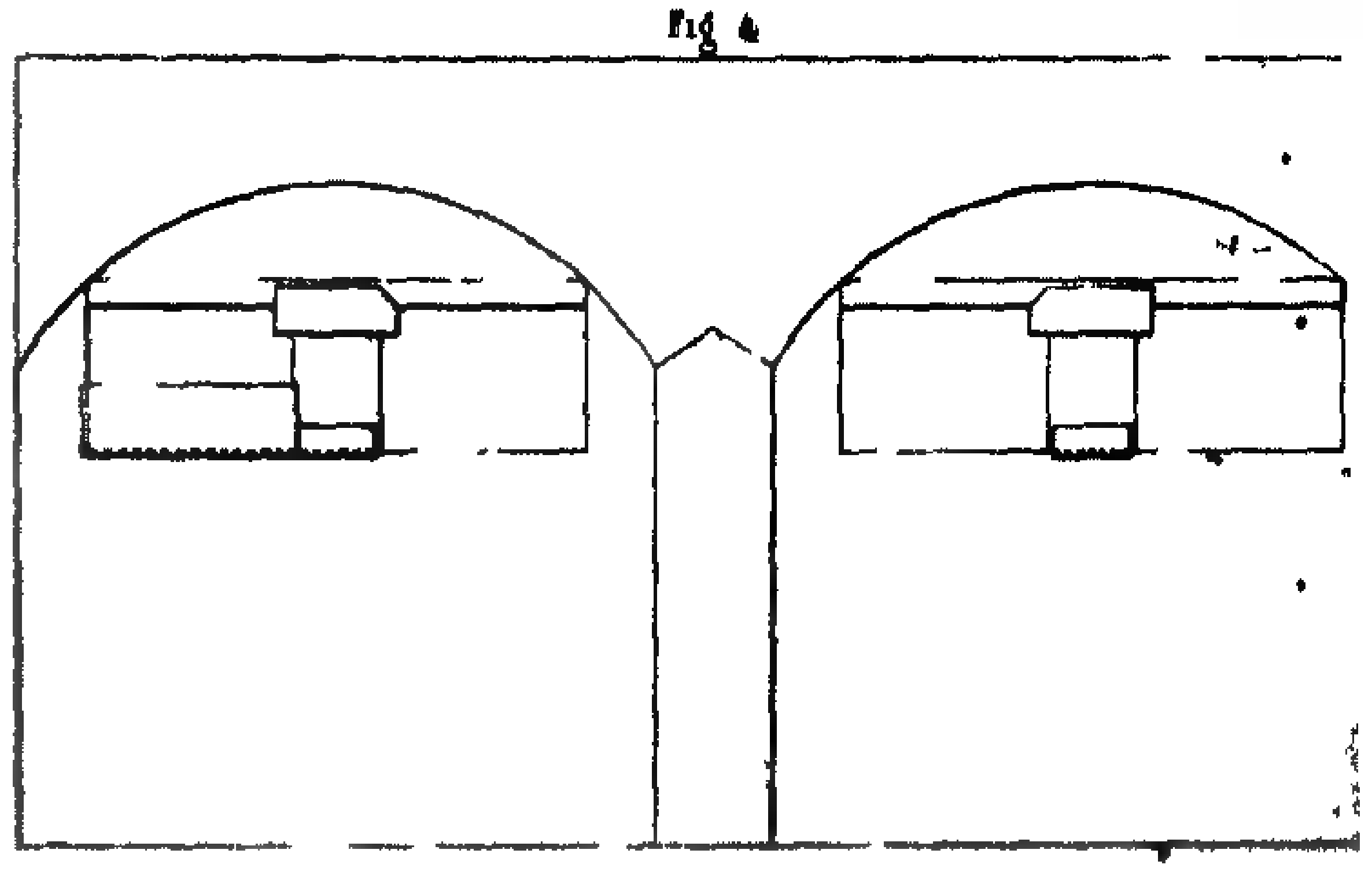


Fig 5

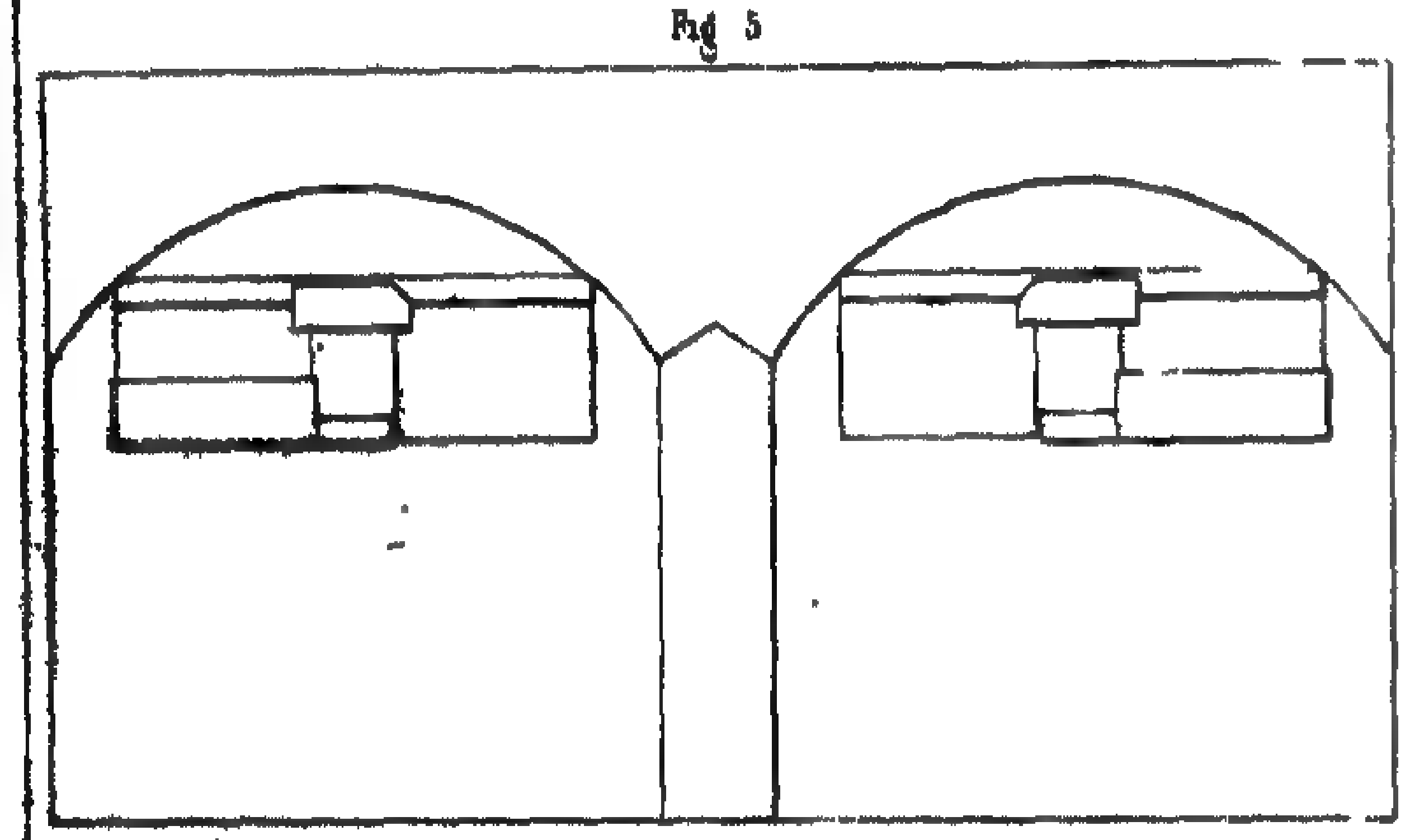


Fig 6

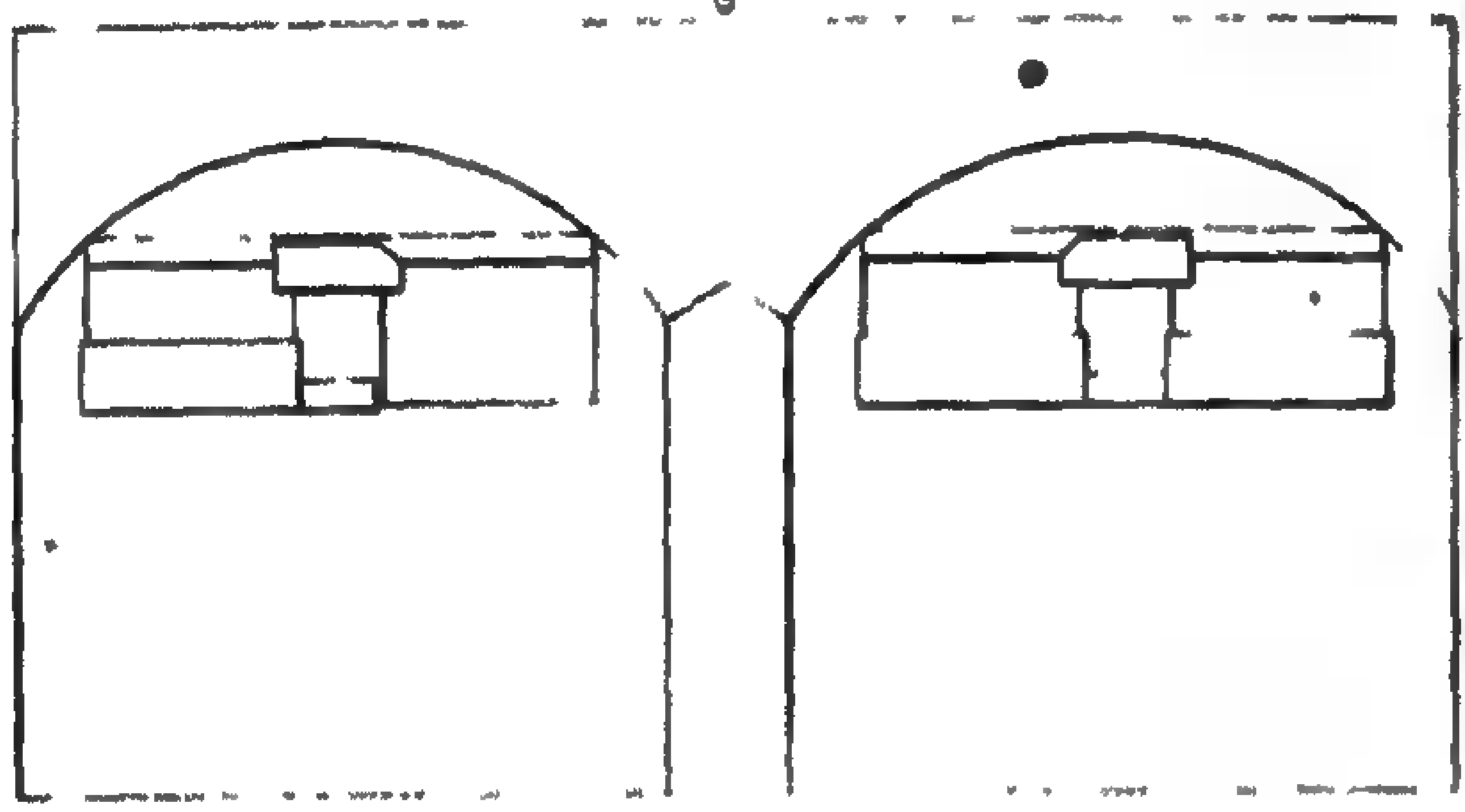


Fig 7

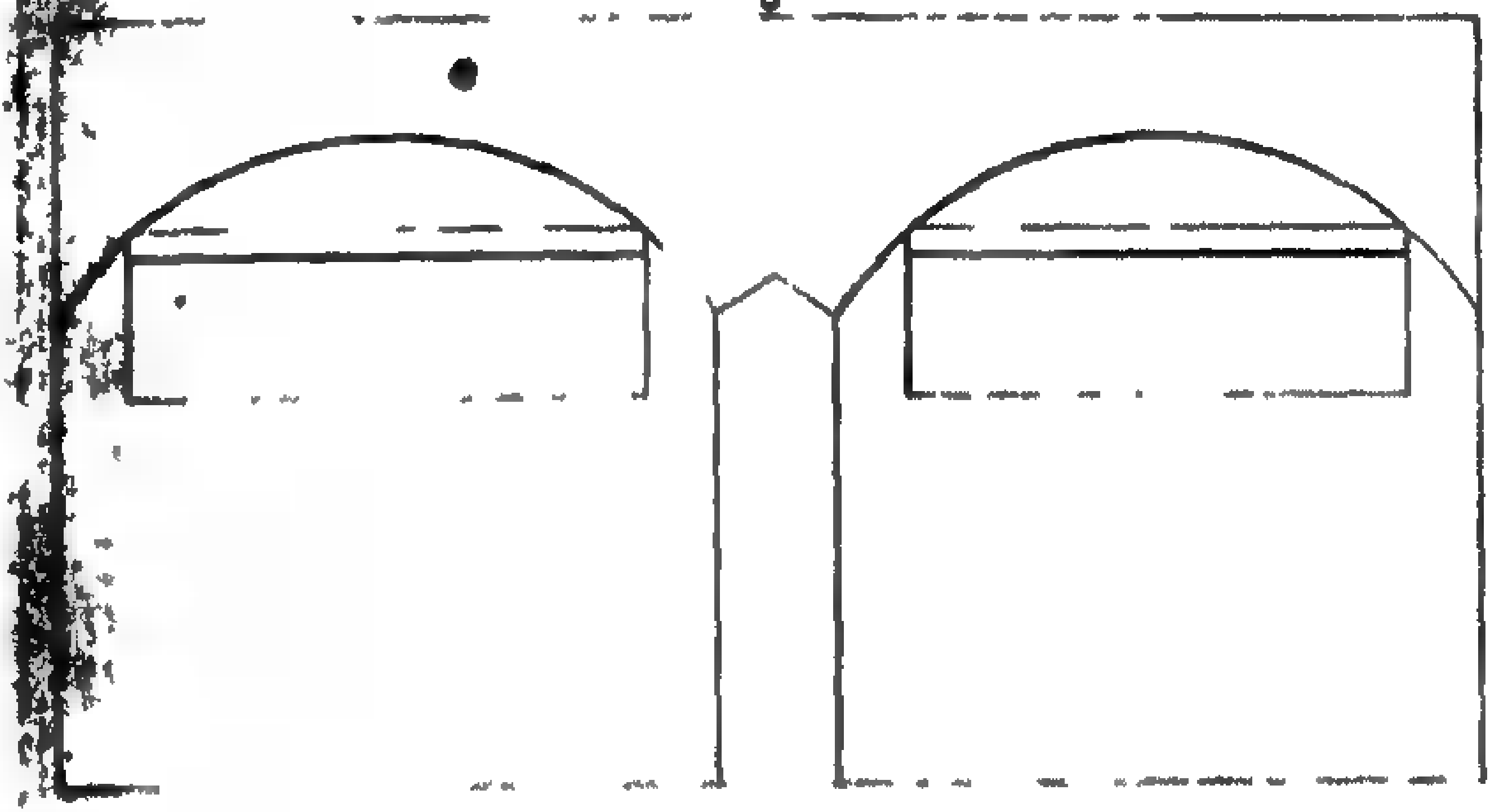


Fig 8

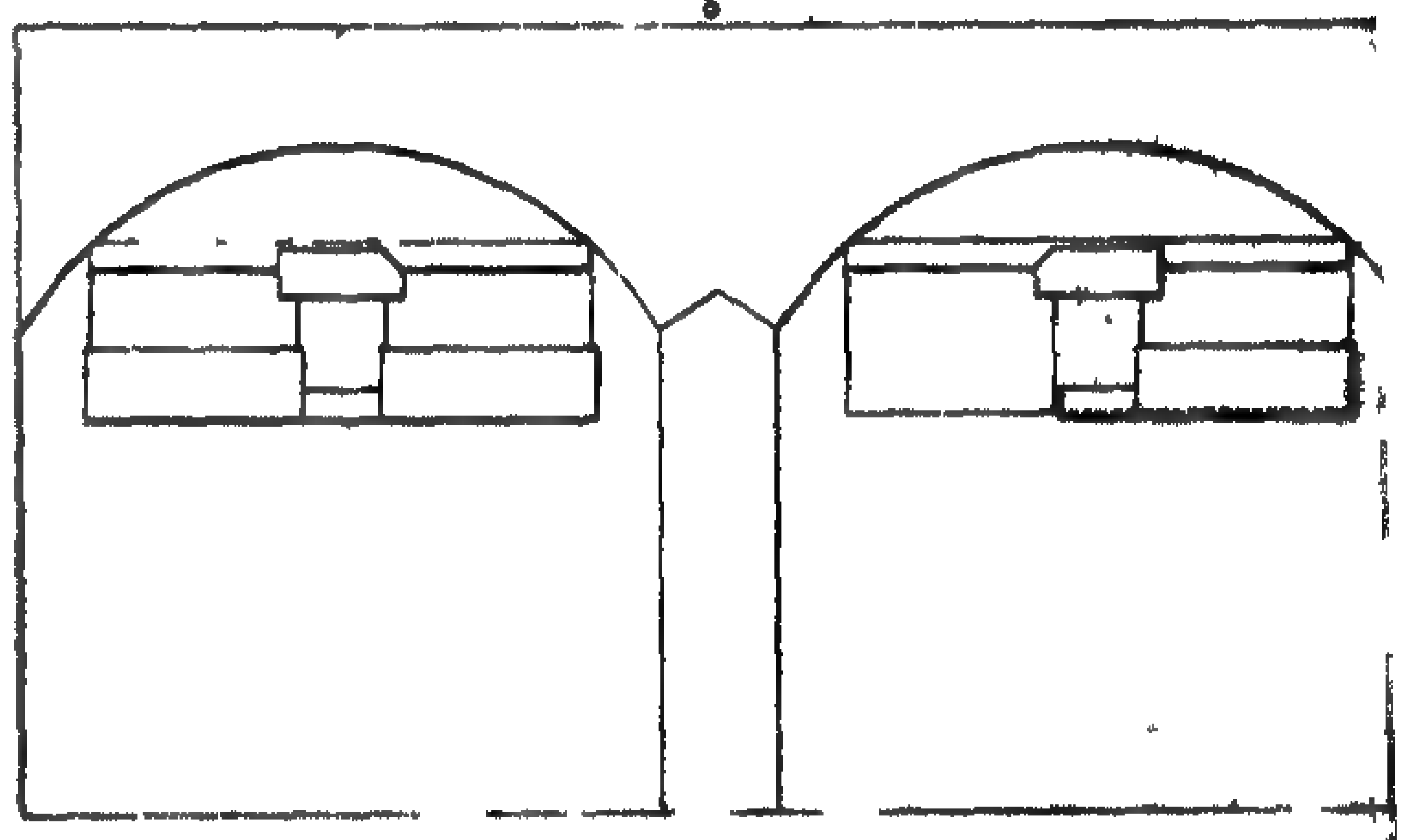


Fig 9

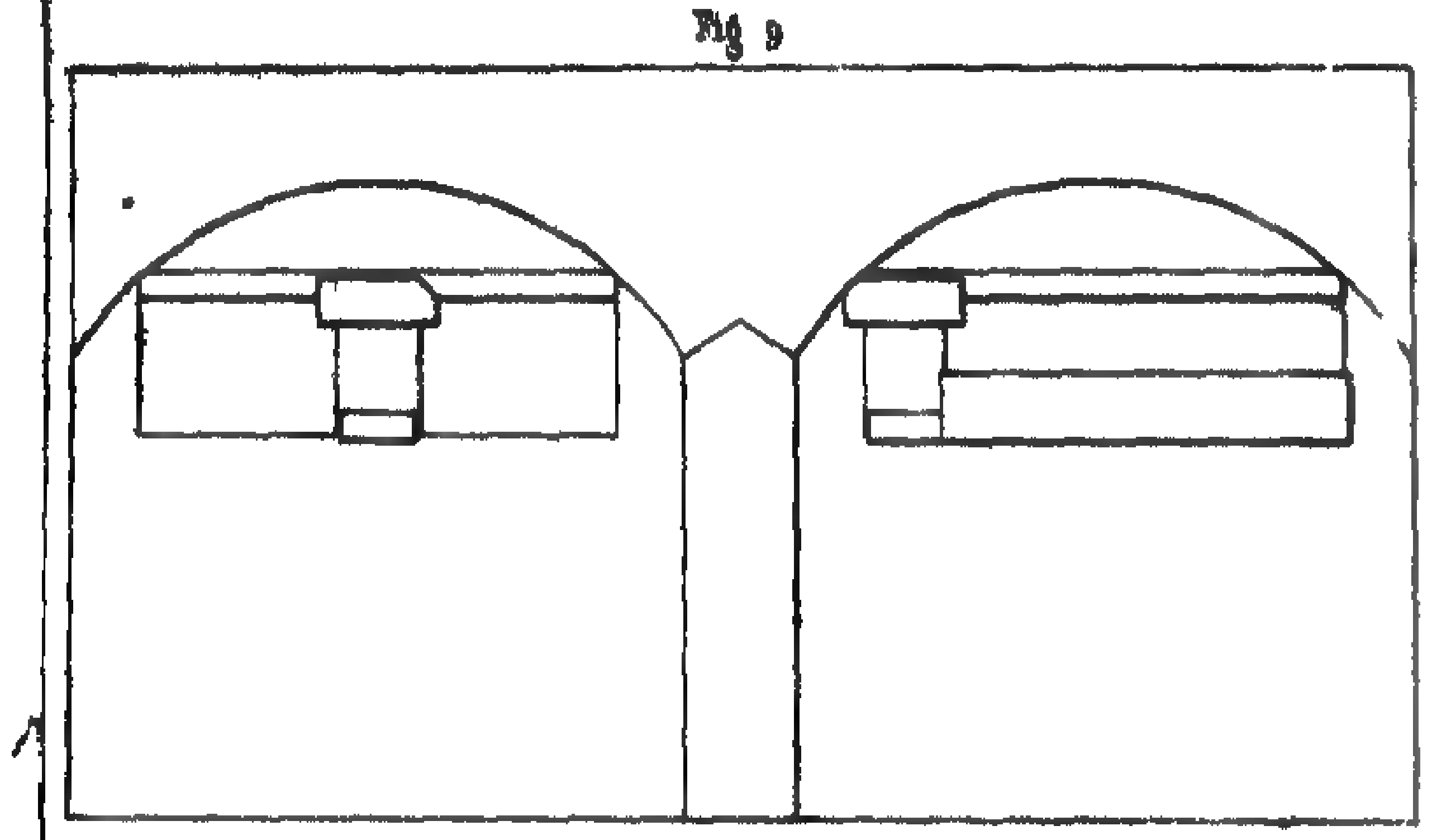


Fig 10

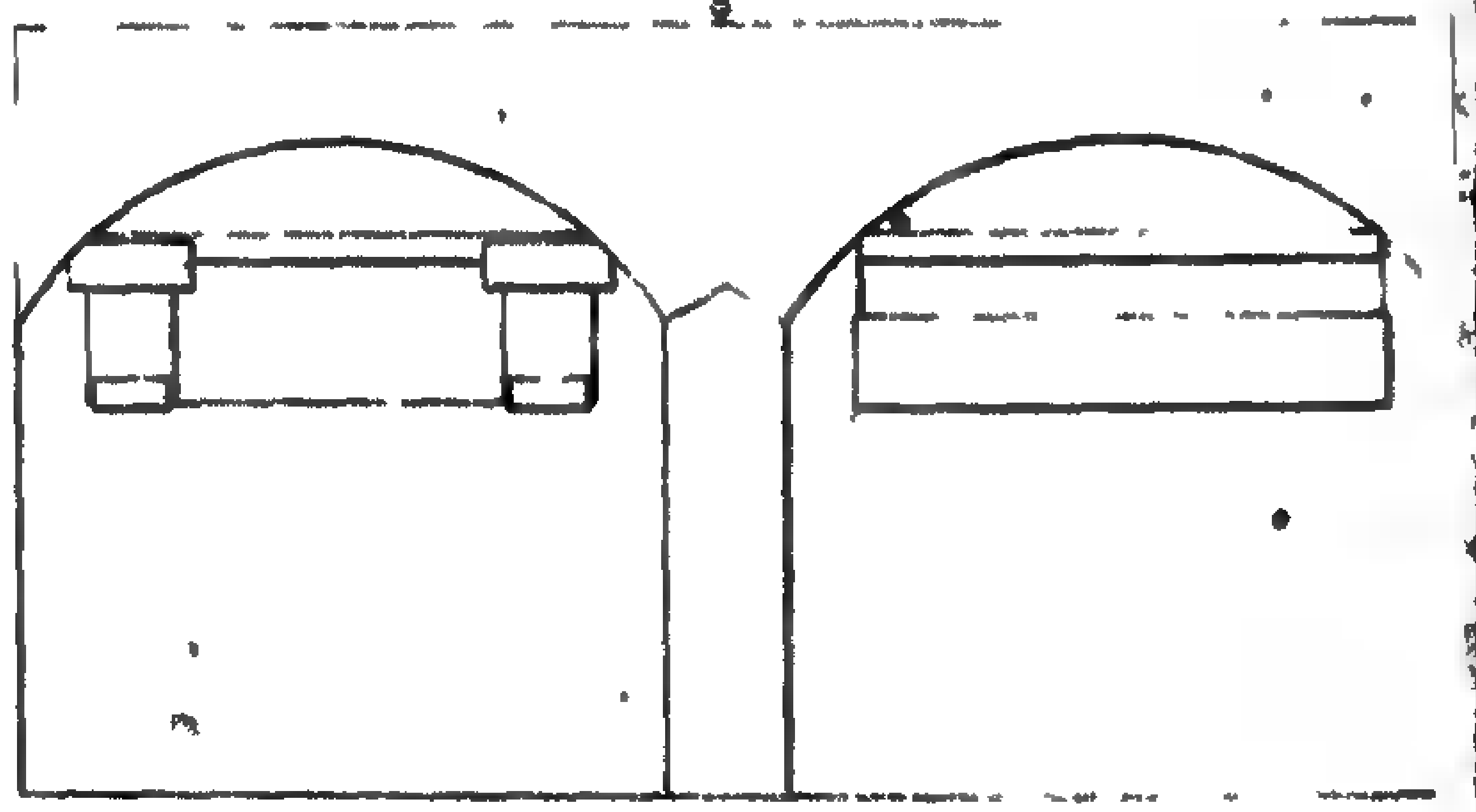


Fig 11

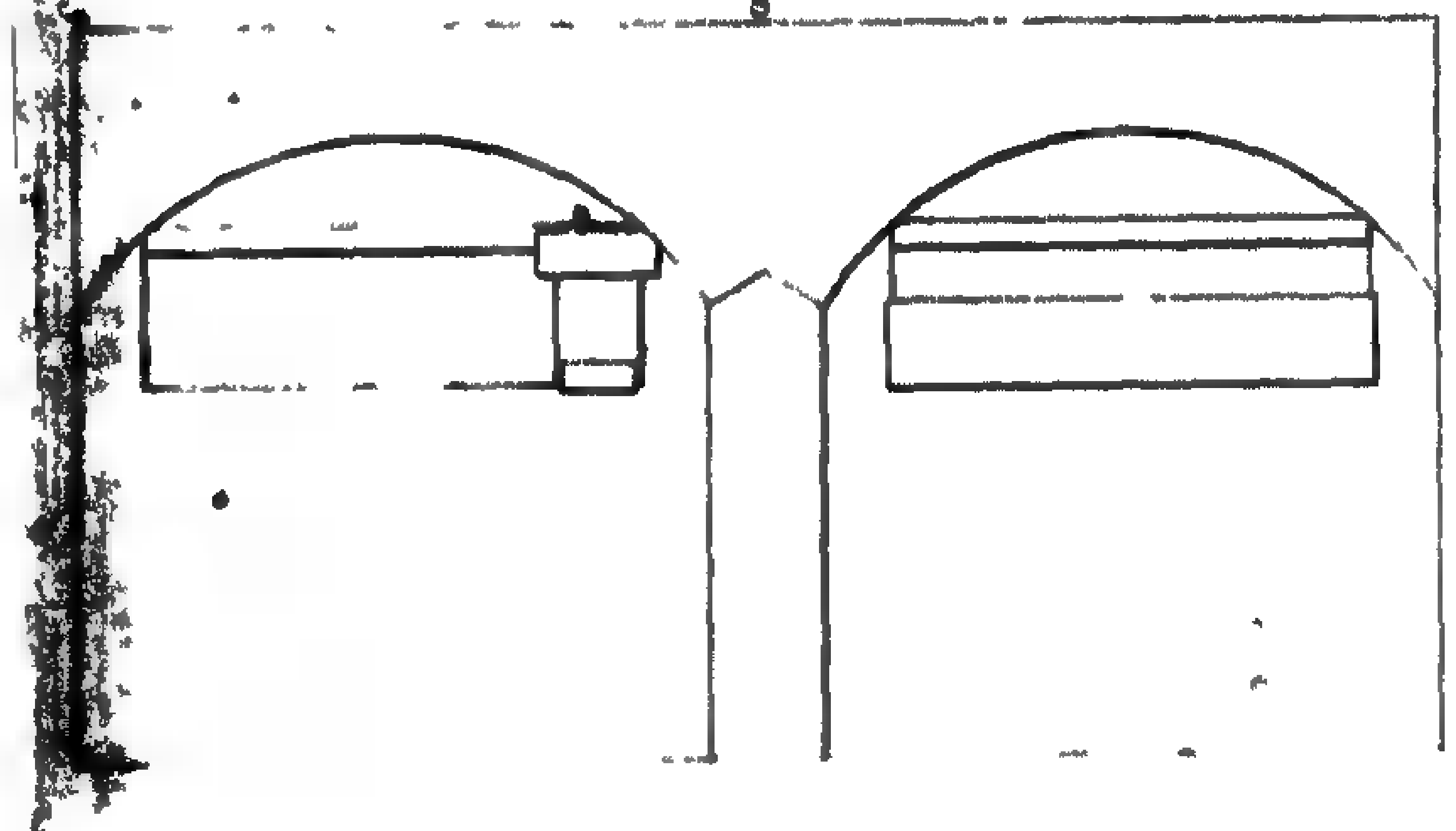


Fig 12

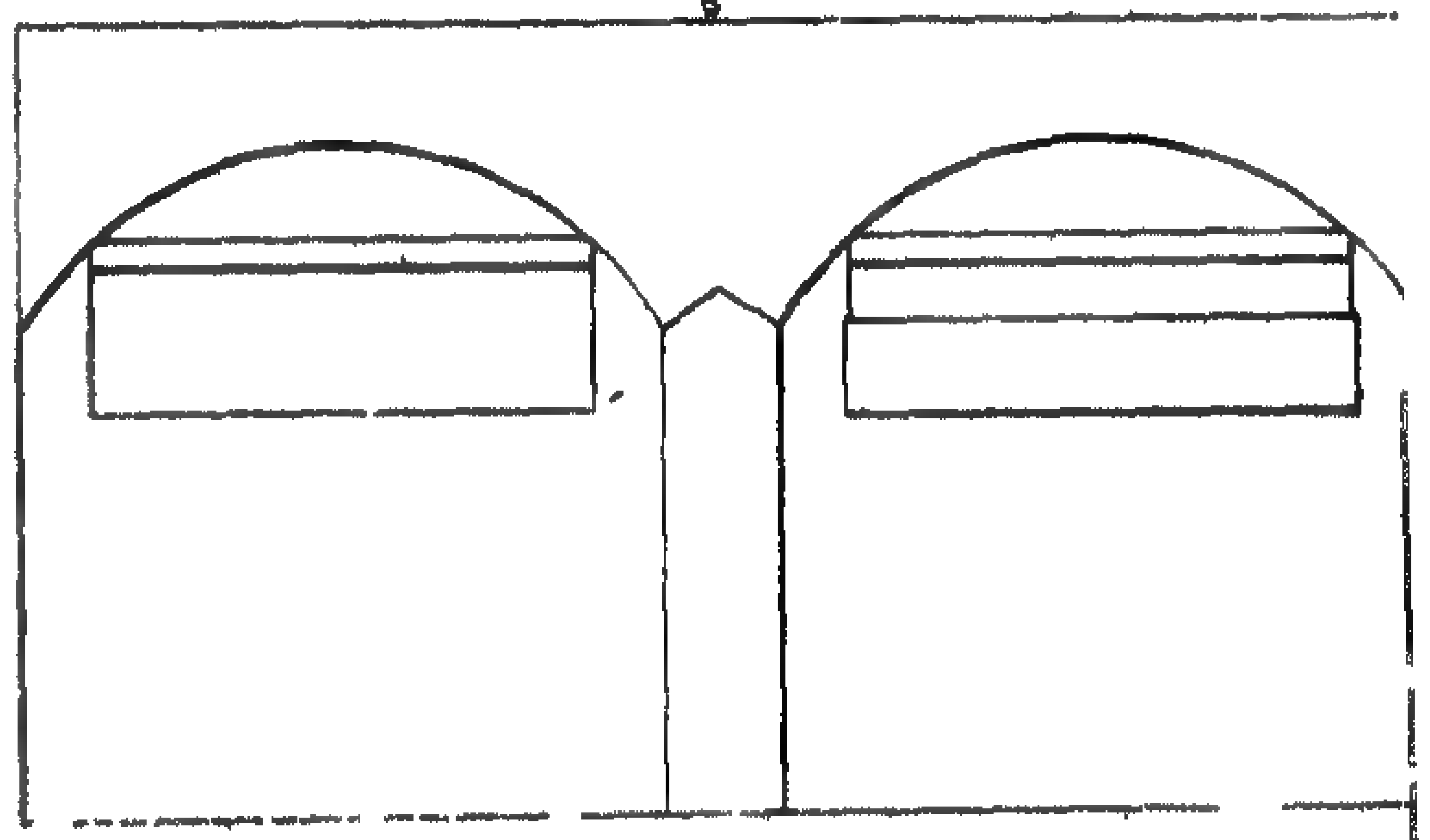


Fig 13

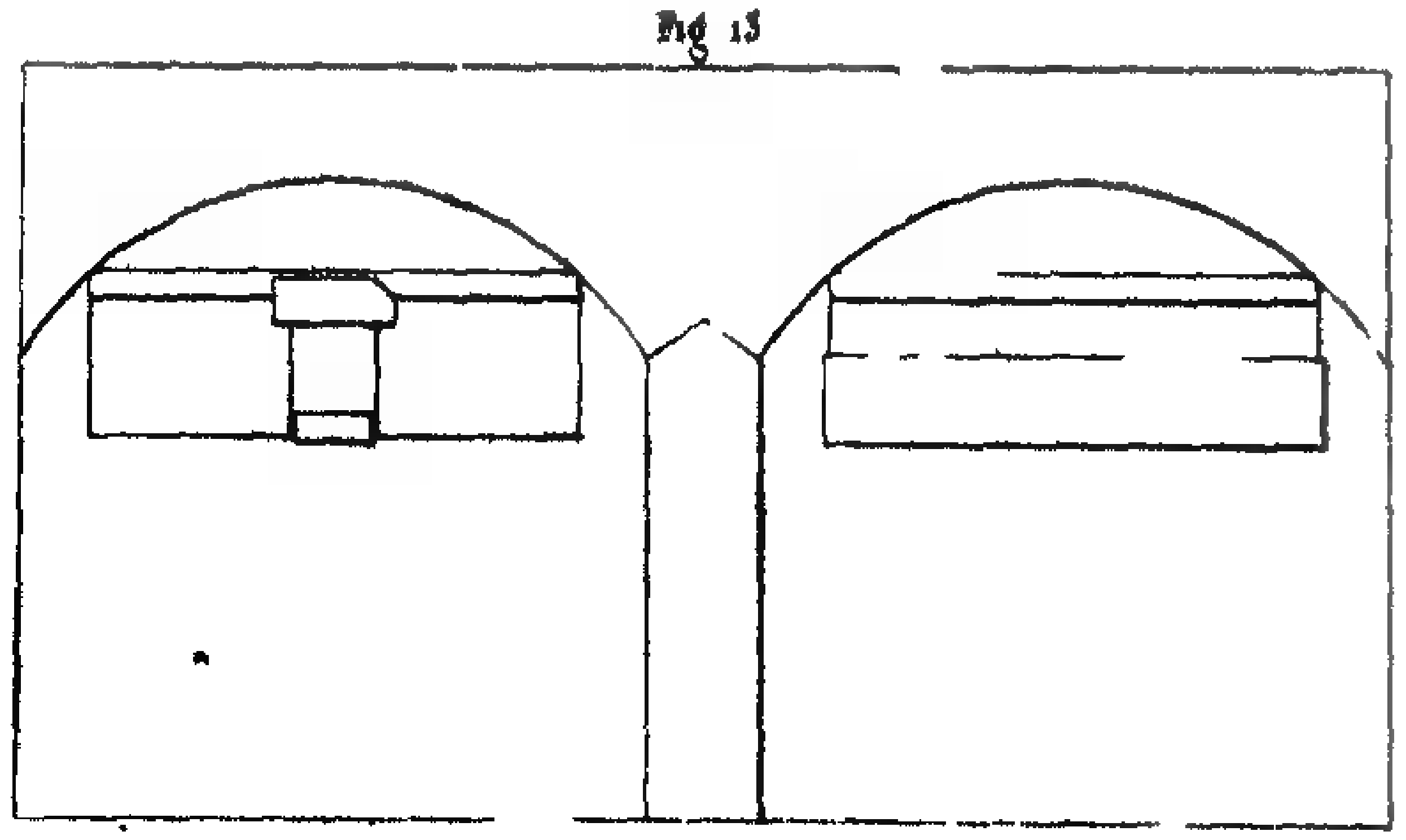


Fig 14

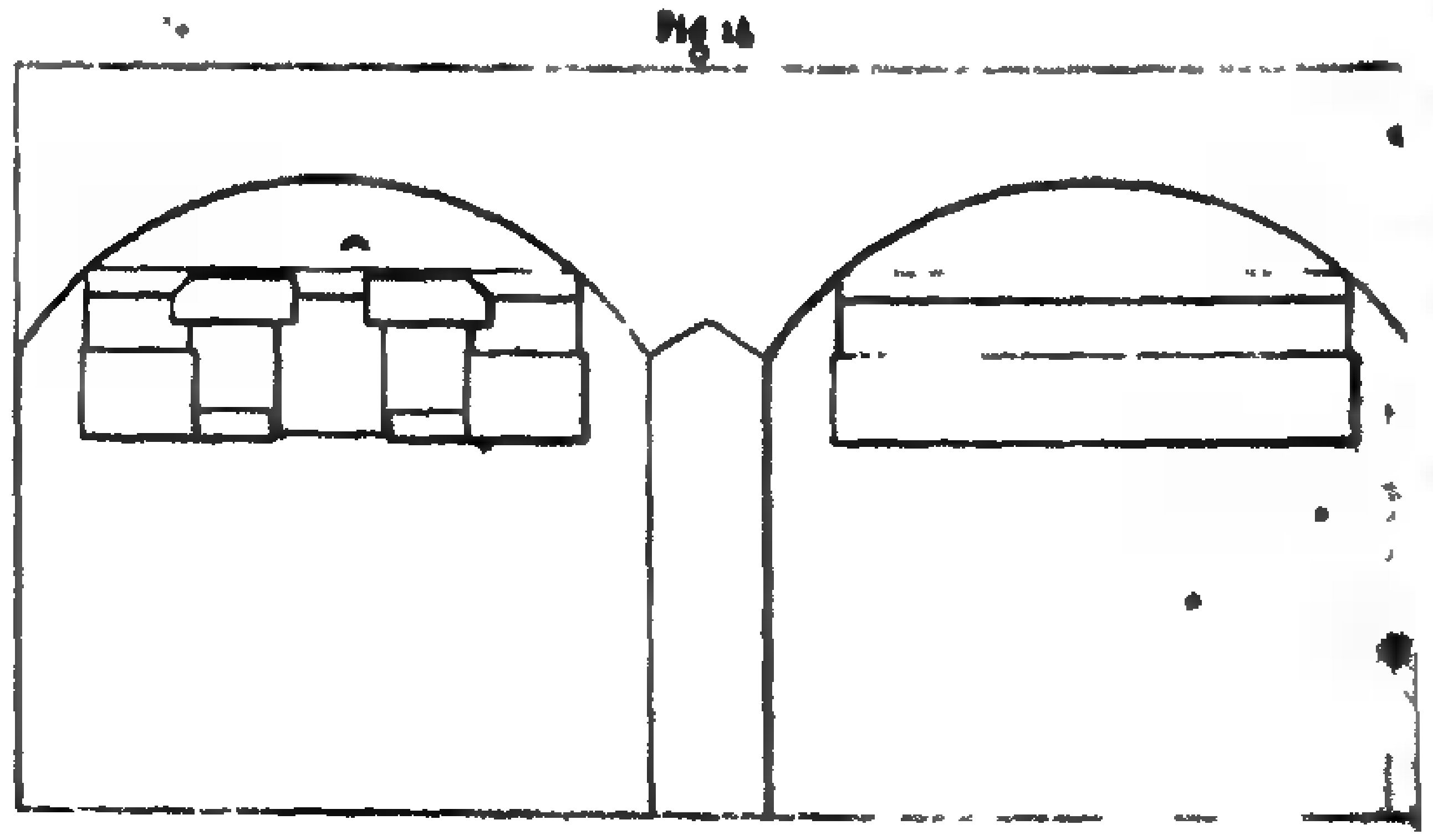


Fig 15

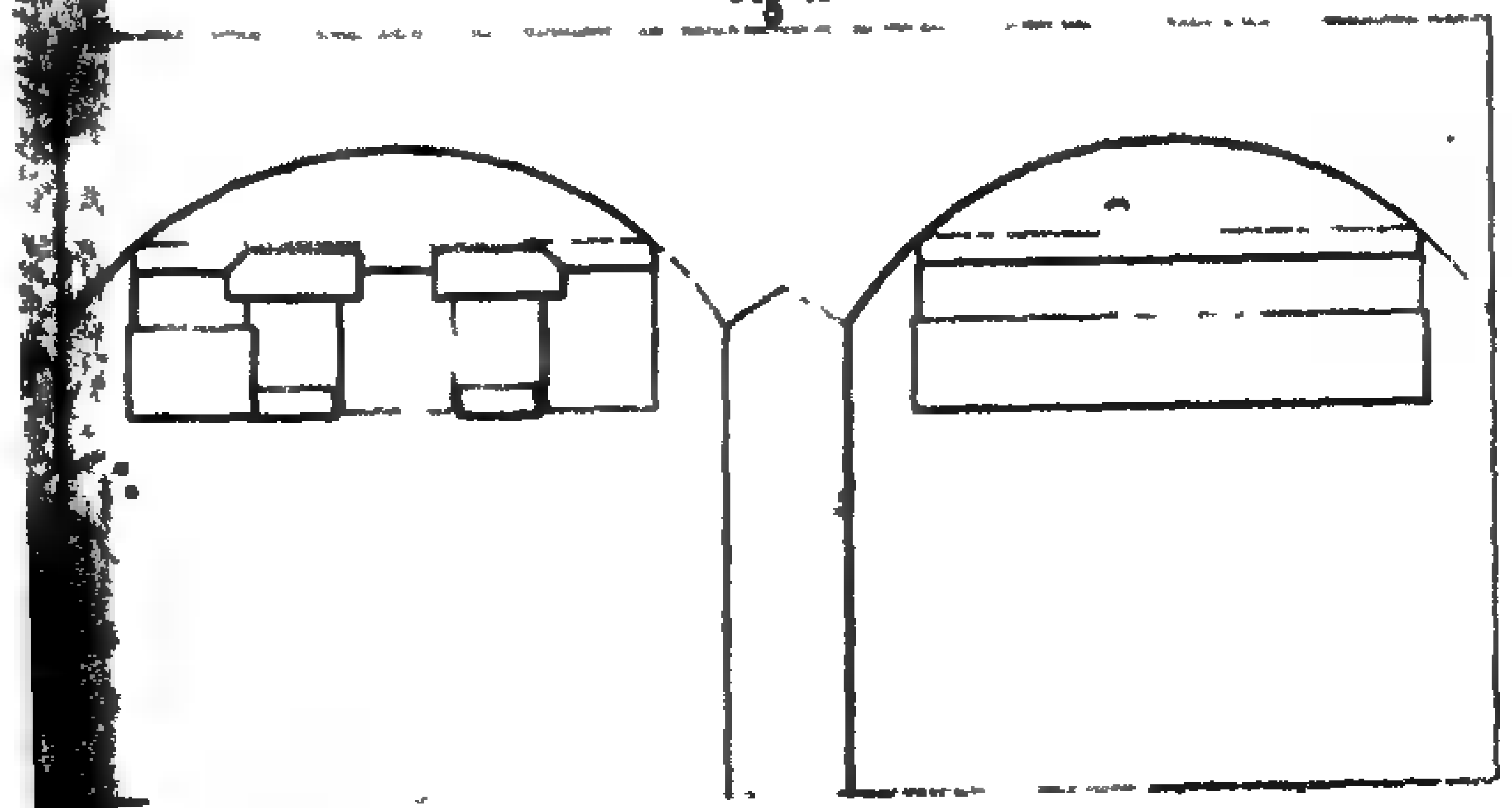
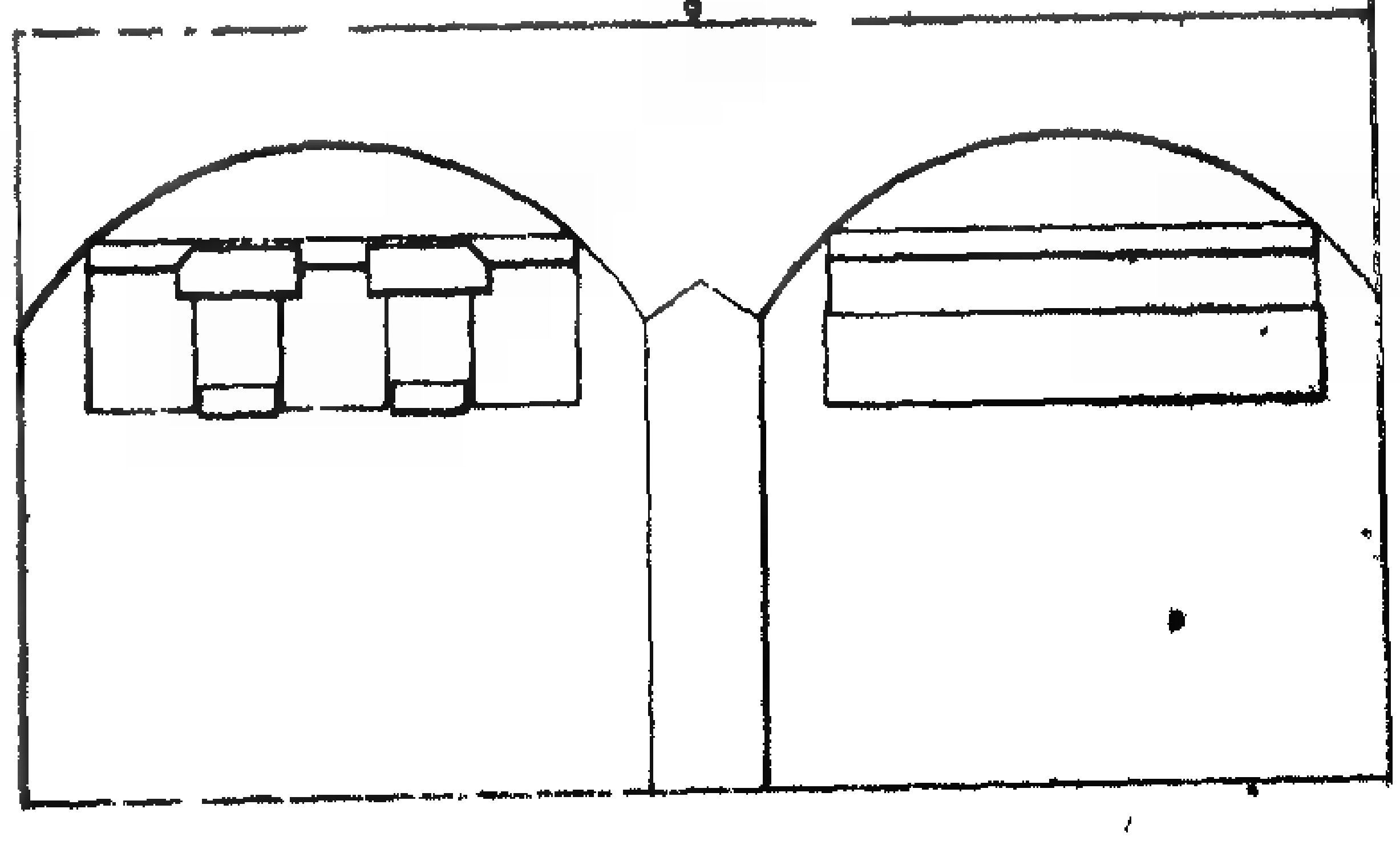


Fig 16



Daniel Haufflaly delt

Fig 1

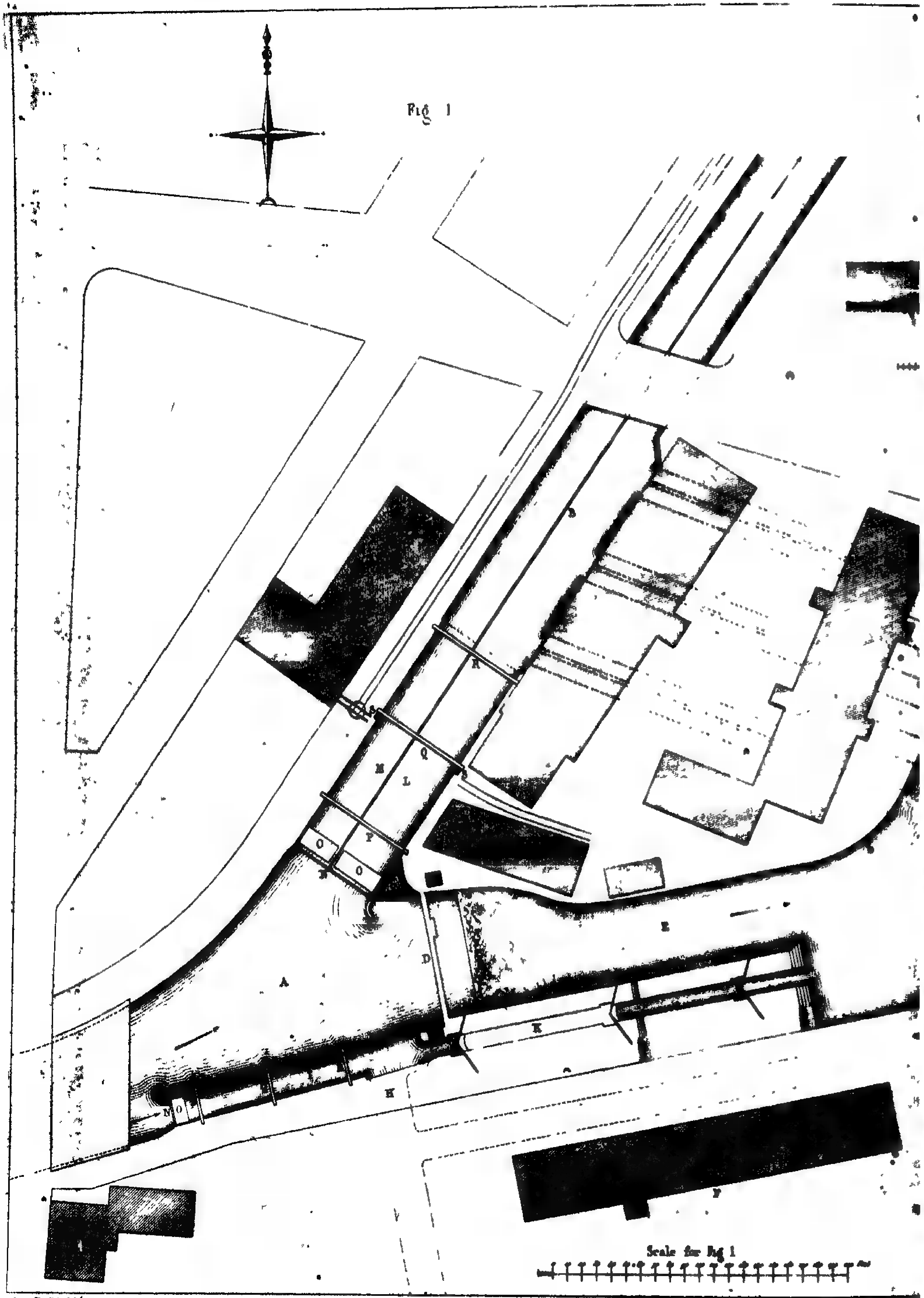


Fig 4

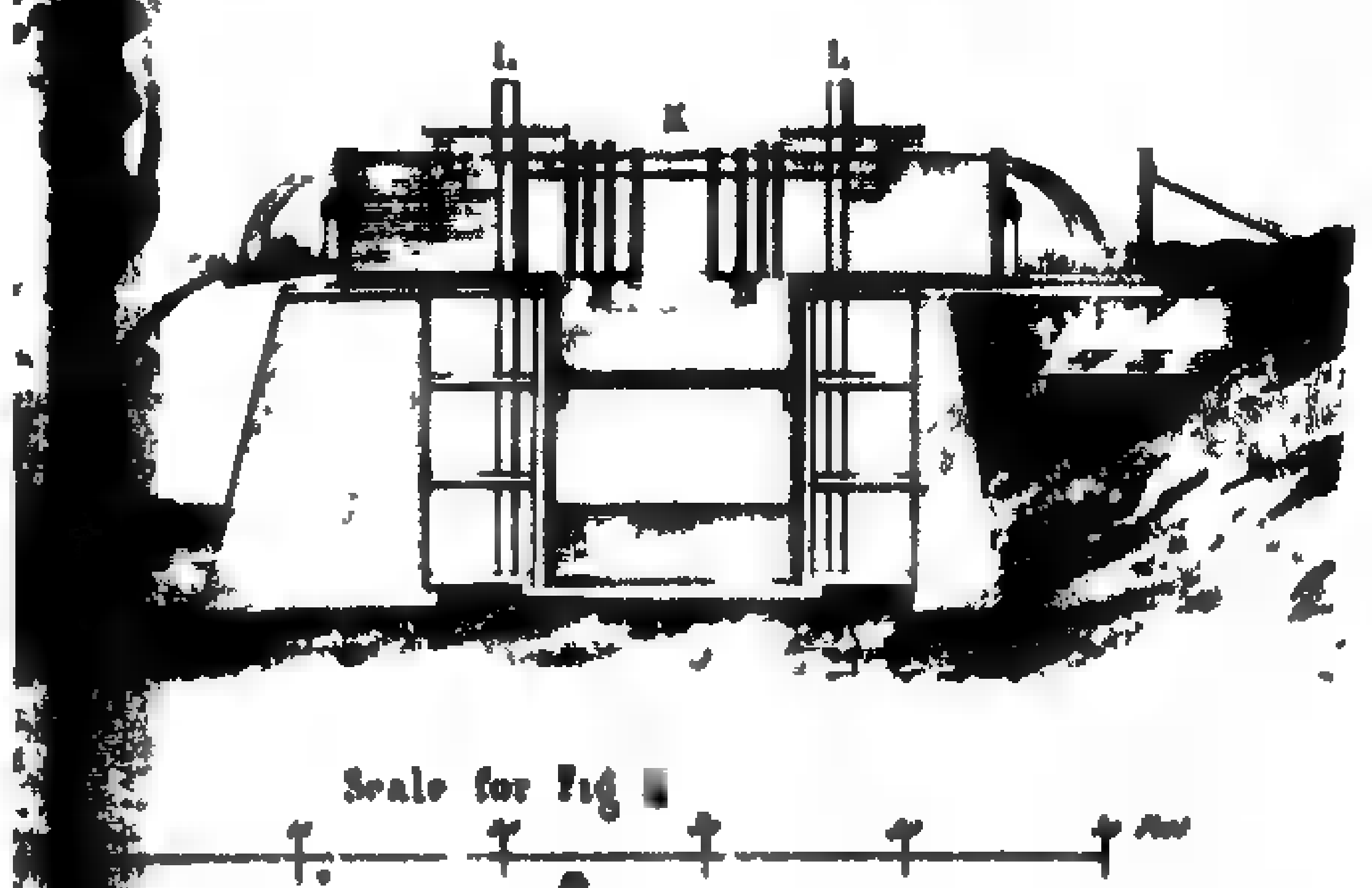


Fig 2

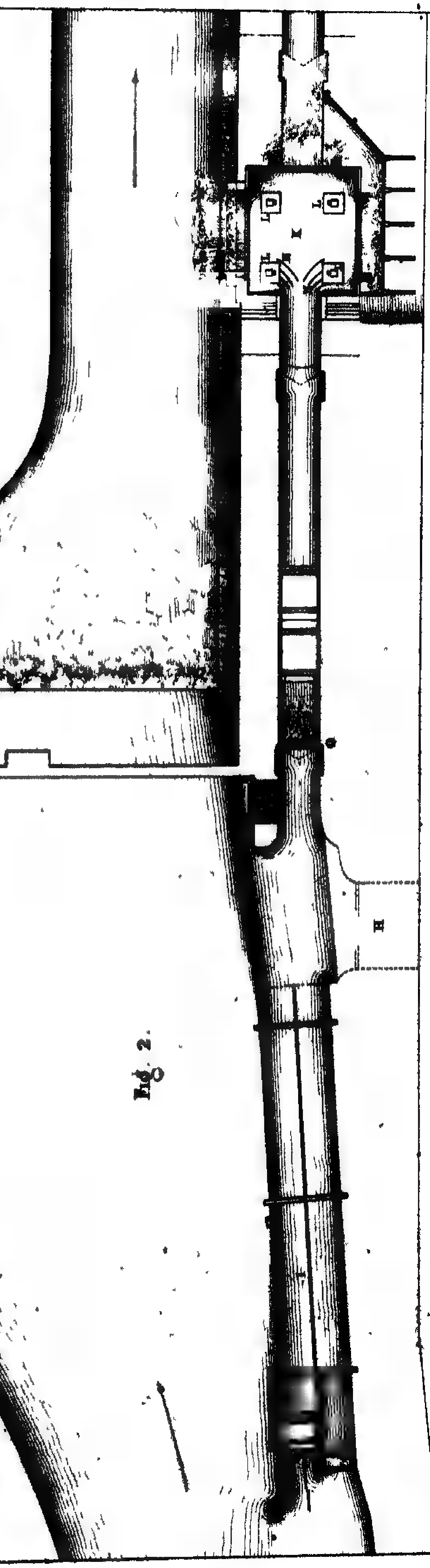
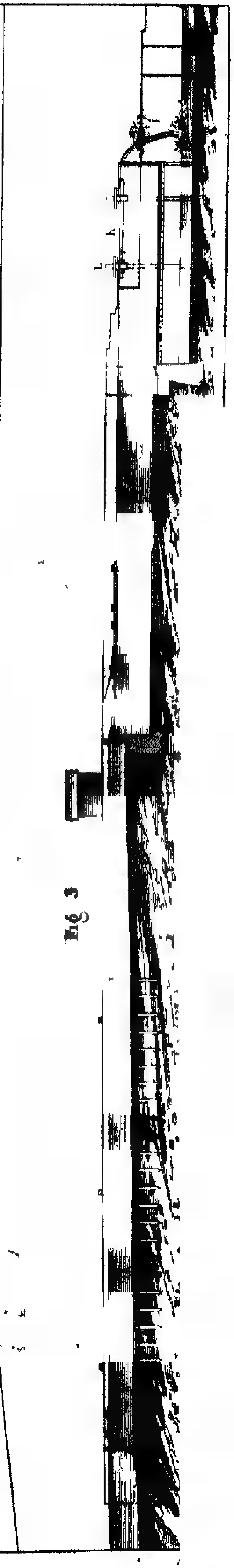


Fig 3



EXPERIMENTS UPON V AT THE LOWER LOCKS

Fig 5.

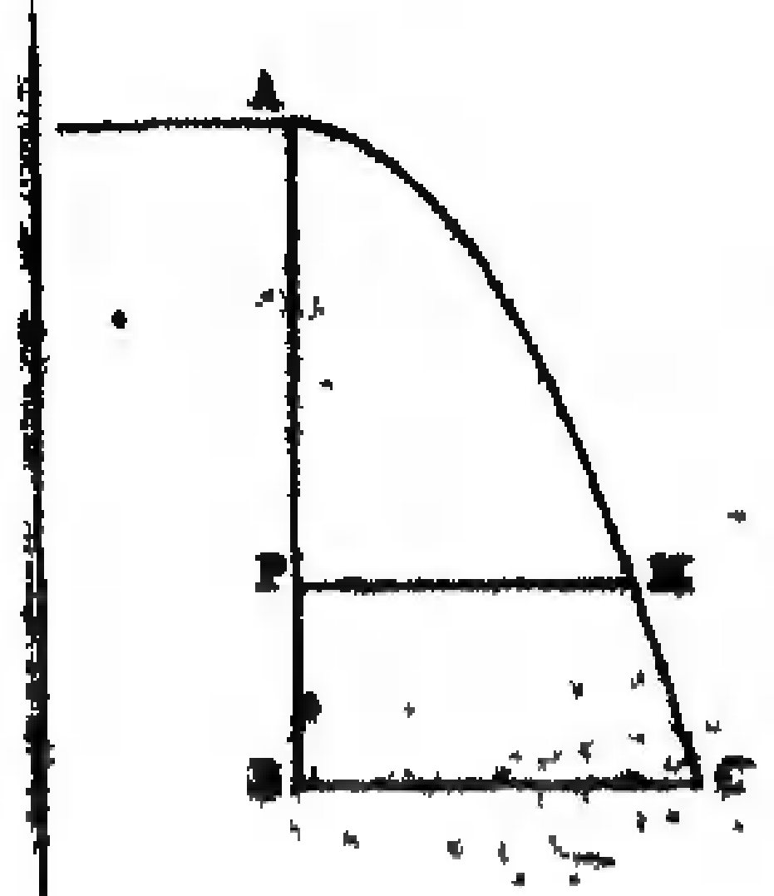


Fig 6

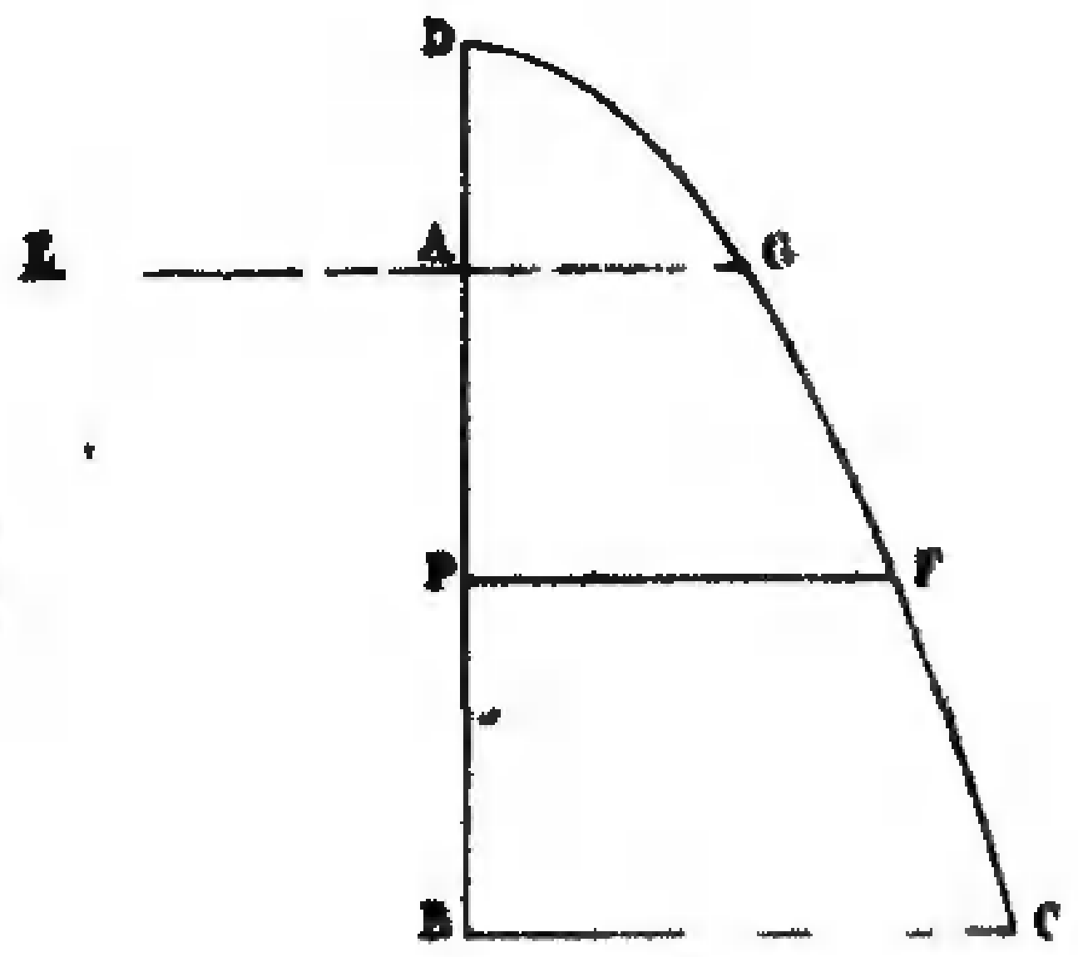


Fig 4

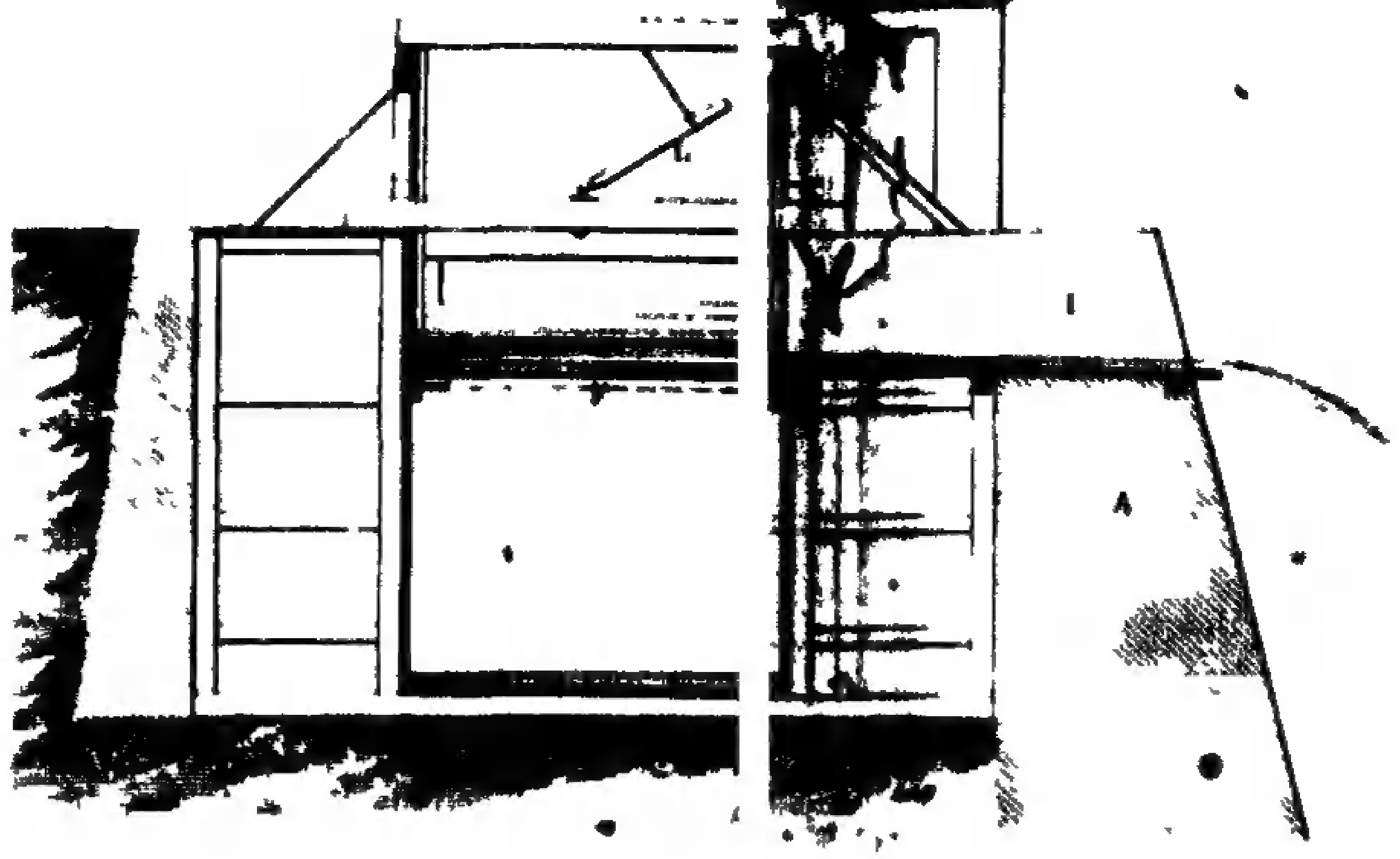
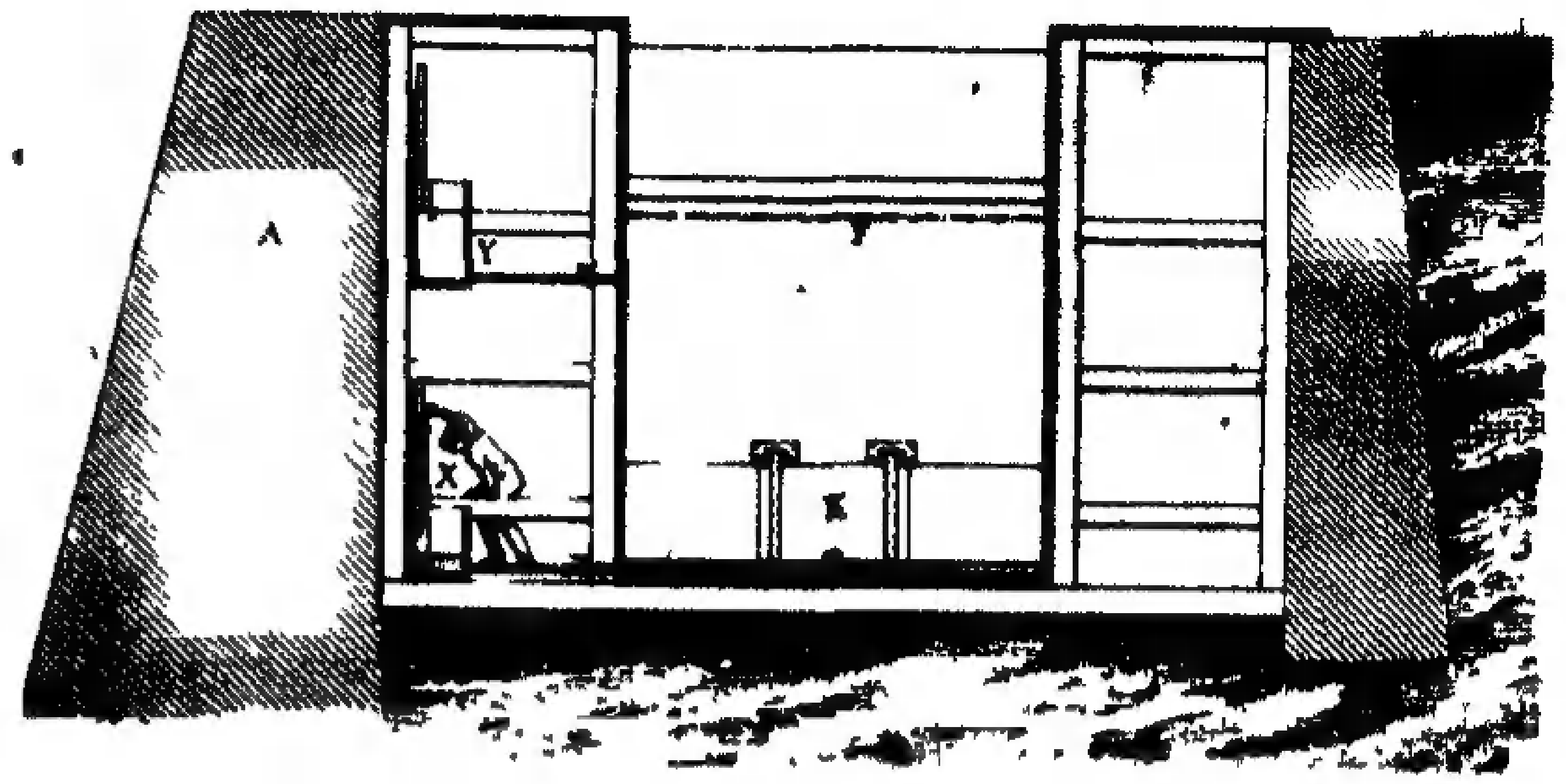


Fig 3



Fig

Fig 1

Fig 5 (full size)
Fig 2
Fig 3
Fig 1
Fig 4
A
B
C
D
E
F
G
H
I
K
L
M
N
Q
R
S
T
U
Scale for Figures 2 3 and 4
Scale for Fig 1

WEIRS EXPERIMENTED AT THE LOWER LOCKS

Pl XIV

Fig 1

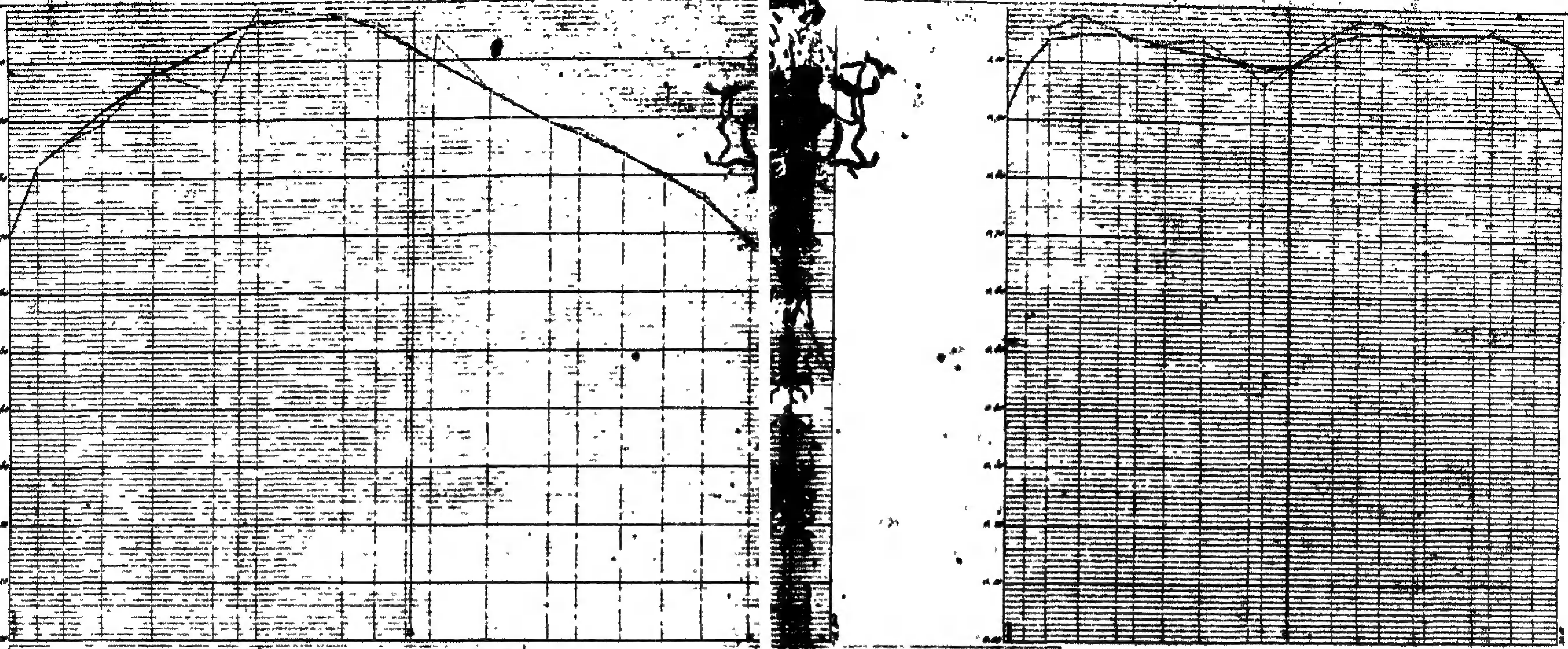


Fig. 3.